Chapter 4: Sequences and Series

EXERCISE 4.1 [PAGES 50 - 51]

Exercise 4.1 | Q 1.1 | Page 50

Verify whether the following sequences are G.P. If so, write t_n : 2, 6, 18, 54, ...

SOLUTION

2, 6, 18, 54, ...
$t_1 = 2$, $t_2 = 6$, $t_3 = 18$, $t_4 = 54$, …

$$\text{Here, } \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 3$$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here. a = 2, r = 3
$t_n = ar^{n-1}$
$\therefore$ $t_n = 2(3^{n-1})$

Exercise 4.1 | Q 1.2 | Page 50

Verify whether the following sequences are G.P. If so, write t_n : 1, – 5, 25, – 125, ...

SOLUTION

1, – 5, 25, – 125, ...
$t_1 = 1$, $t_2 = -5$, $t_3 = 25$, $t_4 = -125$, …

$$\text{Here, } \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = -5$$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.
Here, a = 1, r = – 5
$t_n = ar^{n-1}$
$\therefore$ $t_n = (-5)^{n-1}$

Exercise 4.1 | Q 1.3 | Page 50

Verify whether the following sequences are G.P. If so, write t_n :

$$\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots$$

SOLUTION

$\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \ldots$

$t_1 = \sqrt{5}, t_2 = \frac{1}{\sqrt{5}}, t_3 = \frac{1}{5\sqrt{5}}, t_4 = \frac{1}{25\sqrt{5}}, \ldots$

Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5}$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here, $a = \sqrt{5}, r = \frac{1}{5}$

$t_n = ar^{n-1}$

$\therefore tn = \sqrt{5}\left(\frac{1}{5}\right)^{n-1}$

$= (5)^{\frac{1}{2}}(5)^{1-n}$

$= (5)^{\frac{3}{2}-n}$.

Exercise 4.1 | Q 1.4 | Page 50

Verify whether the following sequences are G.P. If so, write t_n : 3, 4, 5, 6, ...

SOLUTION

3, 4, 5, 6, ...

$t_1 = 3, t_2 = 4, t_3 = 5, t_4 = 6, \ldots$

Here, $\frac{t_2}{t_1} = \frac{4}{3}, \frac{t_3}{t_2} = \frac{5}{4}, \frac{t_4}{t_3} = \frac{6}{5}$

Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

$\therefore$ the given sequence is not a geometric progression.

Exercise 4.1 | Q 1.5 | Page 50

Verify whether the following sequences are G.P. If so, write t_n : 7, 14, 21, 28, ...

SOLUTION

7, 14, 21, 28, ...

$t_1 = 7$, t_2 14, $t_3 = 21$, $t_4 = 28$, ...

Here, $\frac{t_2}{t_1} = 2, \frac{t_3}{t_2} = \frac{3}{2}, \frac{t_4}{t_3} = \frac{4}{3}$

Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

$\therefore$ the given sequence is not a geometric progression.

Exercise 4.1 | Q 2.1 | Page 50

For the G.P., if $r = \frac{1}{3}$, $a = 9$, find t_7.

SOLUTION

Given, $r = \frac{1}{3}$, $a = 9$

$t_n = ar^{n-1}$

$\therefore t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1}$

$= \frac{9}{3^6}$

$= \frac{1}{81}$.

Exercise 4.1 | Q 2.2 | Page 50

For the G.P., if $a = \frac{7}{243}, r = \frac{1}{3}$, find t_3.

SOLUTION

Given, a = $\frac{7}{243}, r = \frac{1}{3}$

$t_n = ar^{n-1}$

$\therefore t_3 = \frac{7}{243} \times \left(\frac{1}{3}\right)^{3-1}$

$= \frac{7}{243} \times \left(\frac{1}{3}\right)^{2}$

$= \frac{7}{243} \times \frac{1}{9}$

$= \frac{7}{2187}$.

Exercise 4.1 | Q 2.3 | Page 50

For the G.P., if a = 7, r = – 3, find t_6.

SOLUTION

Given, a = 7, r = – 3

$t_n = ar^{n-1}$

$\therefore t_6 = 7 \times (-3)^{6-1}$

$= 7 \times (-3)^5$

$= 7 \times (-243)$

= – 1701.

Exercise 4.1 | Q 2.4 | Page 50

For the G.P., if a = $\frac{2}{3}$, $t_6 = 162$, find r.

SOLUTION

Given, a = $\frac{2}{3}$, $t_6 = 162$

$t_n = ar^{n-1}$

$\therefore t_6 = \left(\frac{2}{3}\right)(r^{6-1})$

$\therefore 162 = \frac{2}{3} r^5$

$\therefore r^5 = 162 \times \frac{3}{2}$

$\therefore r^5 = 3^5$

$\therefore r = 3.$

Exercise 4.1 | Q 3 | Page 50

Which term of the G. P. 5, 25, 125, 625, … is 5^{10}?

SOLUTION

Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$

$t_n = ar^{n-1}$

$\therefore 5^{10} = 5 \times 5^{(n-1)}$

$\therefore 5^{10} = 5^{(1+n-1)}$

$\therefore 5^{10} = 5^n$

$\therefore n = 10$

$\therefore 5^{10}$ is the 10^{th} term of the G.P.

Exercise 4.1 | Q 4 | Page 50

For what values of x, $\frac{4}{3}, x, \frac{4}{27}$ are in G.P.?

SOLUTION

$\frac{4}{3}, x, \frac{4}{27}$ are in geometric progression.

$\therefore \frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\therefore \frac{x}{\frac{4}{3}} = \frac{\frac{4}{27}}{x}$

$\therefore x^2 = \frac{4}{3} \times \frac{4}{27}$

$\therefore x^2 = \frac{16}{81}$

$\therefore x = \pm\frac{4}{9}$

Exercise 4.1 | Q 5 | Page 50

If for a sequence, $t_n = \frac{5^{n-3}}{2^{n-3}}$, so that the sequence is a G. P. Find its first term and the common ratio.

SOLUTION

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n}$ = constant for all n ∈ N.

Now, $t_n = \frac{5^{n-3}}{2^{n-3}}$

$\therefore t_{n+1} = \frac{5^{n+1-3}}{2^{n+1-3}} = \frac{5^{n-2}}{2^{n-2}}$

$\therefore \frac{t_{n+1}}{t_n} = \frac{5^{n-2}}{2^{n-2}} \times \frac{2^{n-3}}{5^{n-3}}$

$= (5)^{(n-2)-(n-3)} \times (2)^{(n-3)-(n-2)}$

$= (5)^1 \times (2)^{-1}$

$= \frac{5}{2}$

= constant, for all n ∈ N.

∴ the sequence is a G.P. with common ratio (r) = $\frac{5}{2}$

and fiist term = t_1 = $\frac{5^{1-3}}{2^{1-3}}$

= $\frac{5^{-2}}{2^{-2}}$

= $\frac{2^2}{5^2}$

= $\frac{4}{25}$.

Exercise 4.1 | Q 6 | Page 51

Find three numbers in G. P. such that their sum is 21 and sum of their squares is 189.

SOLUTION

Let the three numbers in G. P. be $\frac{a}{r}$, a, ar.

According to the first condition,

$\frac{a}{r} + a + ar$ = 21

$\therefore \frac{1}{r} + 1 + r = \frac{21}{a}$

$\therefore \frac{1}{r} + r = \frac{21}{a} - 1$ ...(i)

According to the second condition,

$\frac{a^2}{r^2} + a^2 + a^2r^2$ = 189

$\therefore \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$

$$\therefore \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1 \quad ...(ii)$$

On squaring equation (i), we get

$$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore \left(\frac{189}{a^2} - 1\right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1 \quad ...[\text{From (ii)}]$$

$$\therefore \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$$

$$\therefore 441a^2 - \frac{189}{a^2} - \frac{42}{a} = 0$$

$$\therefore \frac{252}{a^2} = \frac{42}{a}$$

$\therefore$ 252 = 42a

$\therefore$ a = 6

Substituting the value of a in (i), we get

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\therefore \frac{1 + r^2}{r} = \frac{15}{6}$$

$$\therefore \frac{1 + r^2}{r} = \frac{5}{2}$$

$\therefore 2r^2 - 5r + 2 = 0$

$\therefore 2r^2 - 4r - r + 2 = 0$

$\therefore (2r - 1)(r - 2) = 0$

$$\therefore r = \frac{1}{2} \text{ or } 2$$

When a 6, $r = \frac{1}{2}$.

$\frac{a}{r}$ = 12, a = 6, ar = 3

When a = 6, r = 2

$\frac{a}{r}$ = 3, a = 6, ar = 12

∴ the three numbers are 12, 6, 3 or 3, 6, 12.

Exercise 4.1 | Q 7 | Page 51

Find four numbers in G. P. such that sum of the middle two numbers is 10/3 and their product is 1.

SOLUTION

Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}$, ar, ar^3.

According to the second condition,

$$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)(ar^3) = 1$$

∴ $a^4 = 1$

∴ a = 1

According to the first condition,

$$\frac{a}{r} + ar \frac{10}{3}$$

$$\therefore \frac{1}{r} + (1)r = \frac{10}{3}$$

$$\therefore \frac{1 + r^2}{r} = \frac{10}{3}$$

∴ $3 + 3r^2 = 10r$

∴ $3r^2 - 10r + 3 = 0$

∴ $(r - 3)(3r - 1) = 0$

$\therefore$ r = 3 or r = $\frac{1}{3}$

When = 3, a = 1

$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}, \frac{a}{r} = \frac{1}{3}$, ar = 1(3) = 3 and ar^3 = 1(3)^3 = 27

When r = $\frac{1}{3}$, a = 1

$$\frac{a}{r^3} = \frac{1}{\left(\frac{1}{3}\right)^3} = 27, \frac{a}{r} = \frac{1}{\left(\frac{1}{3}\right)} = 3,$$

$$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3} \text{ and } r^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$\therefore$ the four numbers in G.P.are

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Exercise 4.1 | Q 8 | Page 51

Find five numbers in G. P. such that their product is 1024 and fifth term is square of the third term.

SOLUTION

Let the five numbers in G. P. be

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

According to the given conditions,

$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2$ = 1024

$\therefore$ a^5 = 45

$\therefore$ a = 4 ...(i)

Also, $ar^2 = a^2$

$\therefore r^2 = a$

$\therefore r^2 = 4$ ...[From (i)]

$\therefore r = \pm 2$

When a = 4, r = 2

$\frac{a}{r^2} = 1, \frac{a}{r} = 2$, a = 4, ar = 8, $ar^2 = 16$

When a = 4, r = – 2

$\frac{a}{r^2} = 1, \frac{a}{r} = -2$, a = 4, ar = – 8, $ar^2 = 16$

When a = 4, r = – 2

$\frac{a}{r^2} = 1, \frac{a}{r} = -2$, a = 4, ar = – 8, $ar^2 = 16$

$\therefore$ the five numbers in G.P. are

1, 2, 4, 8, 16 or – 2, 4, – 8, 16.

Exercise 4.1 | Q 9 | Page 51

The fifth term of a G. P. is x, eighth term of the G. P. is y and eleventh term of the G. P. is z. Verify whether $y^2 = xz$.

SOLUTION

Given, $t_5 = x$, $t_8 = y$, $t_{11} = z$
Since, $t_n = ar^{n-1}$
$\therefore t_5 = ar^4$, $t8 = ar^7$, $t11 = ar^{10}$
Consider,
L.H.S. = $y^2 = (t_8)^2 = (ar_7)^2 = a^2r^{14}$
R.H.S. = = $xz = t_5.t_{11} = ar^4.ar^{10} = a^2r^{14}$
$\therefore$ L.H.S. = R.H.S.
$\therefore y^2 = xz$.

Exercise 4.1 | Q 10 | Page 51

If p, q, r, s are in G. P., show that p + q, q + r, r + s are also in G. P.

SOLUTION

p, q, r, s are in G.P.

$\therefore \frac{q}{p} = \frac{r}{q} = \frac{s}{r}$

Let $\frac{q}{p} = \frac{r}{q} = \frac{s}{r}$ =

$\therefore$ q = pk, r = qk, s = rk

We have to prove that p + q, q + r, r + s are in G.P.

i.e. to prove that $\frac{q+r}{p+q} = \frac{r+s}{q+r}$

L.H.S. = $\frac{q+r}{p+q} = \frac{q+qk}{p+pk}\frac{q(1+k)}{p(1+k)} = \frac{q}{p}$ = k

R.H.S. = $\frac{r+s}{q+r} = \frac{r+rk}{q+\ qk}\frac{r(1+k)}{q(1+k)} = \frac{r}{q}$ = k

$\therefore \frac{q+r}{p+q} = \frac{r+s}{q+r}$

$\therefore$ p + q, q + r, r + s are in G.P.

EXERCISE 4.2 [PAGES 54 - 55]

Exercise 4.2 | Q 1.1 | Page 54

For the following G.P.'s, find S_n: 3, 6, 12, 24, ...

SOLUTION

3, 6, 12, 24, ...

Here, a = 3, r = $\frac{6}{3}$ = 2 > 1

$S_n = \frac{a(r^n - 1)}{r - 1}$, for r > 1

$\therefore S_n = \dfrac{3(2^n - 1)}{2 - 1}$

$\therefore S_n = 3(2^n - 1)$

Exercise 4.2 | Q 1.2 | Page 54

For the following G.P.'s, find S_n: p, q, $\dfrac{q^2}{p}, \dfrac{q^3}{p^2}, \ldots$

SOLUTION

p, q, $\dfrac{q^2}{p}, \dfrac{q^3}{p^2}, \ldots$

Here, a = p, r = $\dfrac{q}{p}$

Let $\dfrac{q}{p} < 1$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$, for r < 1

$\therefore S_n = \dfrac{p\left[1 - \left(\frac{q}{p}\right)^n\right]}{1 - \frac{q}{p}}$

$\therefore S_n = \dfrac{p^2}{p - q}\left[1 - \left(\dfrac{q}{p}\right)^n\right]$

Let $\dfrac{q}{p} > 1$

$S_n = \dfrac{a(r^n - 1)}{r\ \ 1}$, for r > 1

$\therefore S_n = \dfrac{p\left[\left(\frac{q}{p}\right)^n - 1\right]}{\frac{q}{p} - 1}$

$= \dfrac{p^2}{q - p}\left[\left(\dfrac{q}{p}\right)^n - 1\right]$

Exercise 4.2 | Q 2.1 | Page 54

For a G.P., if a = 2, $r = -\frac{2}{3}$, find S_6.

SOLUTION

$a = 2, r = -\frac{2}{3}$

$S_n = \frac{a(1-r^n)}{1-r}$, for r < 1

$\therefore S_6 = \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{1-\left(-\frac{2}{3}\right)}$

$= \frac{2\left[1-\left(-\frac{2}{3}\right)^6\right]}{\frac{5}{3}}$

$= \frac{6}{5}\left[\frac{729-64}{3^6}\right]$

$= \frac{6}{5}\left[\frac{665}{729}\right]$

$\therefore S_6 = \frac{266}{243}$.

Exercise 4.2 | Q 2.2 | Page 54

For a G.P., if $S_5 = 1023$, r = 4, find a.

SOLUTION

$r = 4, S_5 = 1023$

$S_n = a\left(\frac{r^n-1}{r-1}\right)$, for r > 1

$\therefore S_5 = a\left(\frac{4^5-1}{4-1}\right)$

$\therefore 1023 = a\left(\frac{1024-1}{3}\right)$

$\therefore 1023 = \frac{a}{3}(1023)$

$\therefore\ a = 3.$

Exercise 4.2 | Q 3.2 | Page 54

For a G.P., if sum of first 3 terms is 125 and sum of next 3 terms is 27, find the value of r.

SOLUTION

$S_3 = 125, S_6 = 125 + 24 = 152$

$S_n = a\left(\frac{1-r^n}{1-r}\right)$

$\therefore S_3 = a\left(\frac{1-r^3}{1-r}\right)$

$\therefore 125 = a\left(\frac{1-r^3}{1-r}\right)$ ...(i)

Also, $S_6 = a\left(\frac{1-r^6}{1-r}\right)$

$\therefore 152 = a\left(\frac{1-r^6}{1-r}\right)$ ...(ii)

Dividing (ii) by (i), we get

$$\frac{152}{125} = \frac{1-r^6}{1-r^3}$$

$$\therefore \frac{152}{125} = \frac{\left(1+r^3\right)\left(1-r^3\right)}{\left(1-r^3\right)}$$

$\therefore 1 + r^3 = \frac{152}{125}$

$\therefore r^3 = \frac{152}{125} - 1$

$\therefore r^3 = \frac{27}{125}$

$\therefore r^3 = \left(\frac{3}{5}\right)^3$

$\therefore r = \frac{3}{5}$

Exercise 4.2 | Q 4.1 | Page 55

For a G.P., if $t_3 = 20$, $t_6 = 160$, find S_7.

SOLUTION

$t_3 = 20, t_6 = 160$

$t_n = ar^{n-1}$

$\therefore t_3 = ar^{3-1} = ar^2$

$\therefore ar^2 = 20$

$\therefore a = \frac{20}{r^2}$ ...(i)

Also, $t_6 = ar^5$

$\therefore ar^5 = 160$

$\therefore \left(\frac{20}{r^2}\right)r^5 = 160$ ...[From (i)]

$\therefore r^3 = \frac{160}{20} = 8$

$\therefore r = 2$

Substituting the value of r in (i), we get

$a = \frac{20}{2^2} = 5$

Now, $S_n = \frac{a(r^n - 1)}{r - 1}$, for > 1

$\therefore S_7 = \frac{5(2^7 - 1)}{2 - 1}$

$= 5(128 - 1)$

$= 635.$

Exercise 4.2 | Q 4.2 | Page 55

For a G.P., if $t_4 = 16$, $t_9 = 512$, find S_{10}.

SOLUTION

$t_4 = 16, t_9 = 512$

$t_n = ar^{n-1}$

$\therefore t_4 = ar^{4-1} = ar^3$

$\therefore a = \frac{16}{r^3}$ ...(i)

Also, $t_9 = ar^8$

$\therefore ar^8 = 512$

$\therefore \frac{16}{r^3} \times r^8 = 512$

$\therefore r^5 = 32$

$\therefore r = 2$

Substituting r = 2 in (i), we get

$a = \frac{16}{2^3} = \frac{16}{8} = 2$

Now, $S_n = \dfrac{a(r^n - 1)}{r - 1}$ for r > 1

$\therefore S_{10} = \dfrac{2(2^{10} - 1)}{2 - 1}$

= 2(1024 – 1)

= 2046

Exercise 4.2 | Q 5.1 | Page 55

Find the sum to n terms: 3 + 33 + 333 + 3333 + ...

SOLUTION

S_n = 3 + 33 + 333 + 3333 + ... upto n terms

= 3(1 + 11 + 111 + ... upto n terms)

$= \dfrac{3}{9}$ (9 + 99 + 999 + ... upto n terms)

$= \dfrac{3}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + ... upto n terms]

$= \dfrac{3}{9}$ [(10 + 100 +1000 + ... upto n terms) – (1 + 1 + 1 + ... n times)]

But 10, 100,1000, ... n terms are in G.P.

With a = 10, $r = \dfrac{100}{10} = 10$

$\therefore S_n = \dfrac{3}{9}\left[10\left(\dfrac{10^n - 1}{10 - 1}\right) - n\right]$

$= \dfrac{3}{9}\left[\dfrac{10}{9}(10^n - 1) - n\right]$

$\therefore S_n = \dfrac{1}{27}[10(10^n - 1) - 9n]$

Exercise 4.2 | Q 5.2 | Page 55

Find the sum to n terms: 8 + 88 + 888 + 8888 + …

SOLUTION

S_n = 8 + 88 + 888 + ... upto n terms

= 8(1 + 11 + 111 + ... upto n terms)

$= \frac{8}{9}$ (9 + 99 + 999 + ... upto n terms)

$= \frac{8}{9}$ [(10 – 1) + (100 – 1) + (1000 – 1) + ... upto terms)

$= \frac{8}{9}$[(10 + 100 +1000 + ... upto terms) – (1 + 1 + 1 ... n terms)]

But 10, 100, 1000, ... n terms are in G.P. with

a = 10, r $= \frac{100}{10}$ = 10

$\therefore S_n = \frac{8}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$

$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$\therefore S_n = \frac{8}{81}[10(10^n - 1) - 9n]$.

Exercise 4.2 | Q 6.1 | Page 55

Find the sum to n term: 0.4 + 0.44 + 0.444 + …

SOLUTION

S_n = 0.4 + 0.44 + 0.444 + ... upto n terms

= 4(0.1 + 0.11 + 0.111+ .... upto n terms)

$= \frac{4}{9}$ (0.9 + 0.99 + 0.999 + ... upto n terms)

$= \frac{4}{9}$[(1 – 0.1) + (1 – 0.01) + (1 – 0.001) + ... upto n terms]

$= \frac{4}{9}$[(1 + 1 + 1 ...n times) – (0.01 + 0.01 + 0.001 + ... upto n terms)]

But 0.1, 0.01, 0.001, ... n terms are in G.P.

with a = 0.1, r = $\frac{0.01}{0.1}$ = 0.01

$\therefore S_n = \frac{4}{9}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.01}\right]\right\}$

$\therefore S_n = \frac{4}{9}\left\{n - \frac{0.01}{0.09}[1-(0.1)^n]\right\}$

$\therefore S_n = \frac{4}{9}\left[n - \frac{1}{9}(1-(0.1)n)\right]$

Exercise 4.2 | Q 6.2 | Page 55

Find the sum to n terms: 0.7 + 0.77 + 0.777 + ...

SOLUTION

S_n = 0.7 + 0.77 + 0.777 + ... upto n terms

= 7(0.1 + 0.11 + 0.111+ .... upto n terms)

$= \frac{7}{9}$ (0.9 + 0.99 + 0.999 + ... upto n terms)

$= \frac{7}{9}$[(1 – 0.1) + (1 – 0.01) + (1 – 0.001) + ... upto n terms]

$= \frac{7}{9}$[(1 + 1 + 1 ...n times) – (0.01 + 0.01 + 0.001 + ... upto n terms)]

But 0.1, 0.01, 0.001, ... n terms are in G.P.

with a = 0.1, r = $\frac{0.01}{0.1}$ = 0.01

$$\therefore S_n = \frac{7}{9}\left\{n - 0.1\left[\frac{1-(0.1)^n}{1-0.01}\right]\right\}$$

$$\therefore S_n = \frac{7}{9}\left\{n - \frac{0.01}{0.09}[1-(0.1)^n]\right\}$$

$$\therefore S_n = \frac{7}{9}\left[n - \frac{1}{9}(1-(0.1)n)\right]$$

Exercise 4.2 | Q 7.1 | Page 55

Find the n^{th} terms of the sequences: 0.5, 0.55, 0.555, …

SOLUTION

Let t_1 = 0.5, t_2 = 0.55, t_3 = 0.555 and so on.
t_1 = 0.5
t_2 = 0.55 = 0.5 + 0.05
t_3 = 0.555 = 0.5 + 0.05 + 0.005
∴ t_n = 0.5 + 0.05 + 0.005 + ... upto n terms
But 0.5, 0.05, 0.005, … upto n terms are in
G.P. with a = 0.5 and r = 0.1
∴ t_n = the sum of first n terms of the G.P.

$$\therefore t_n = 0.5\left\{\frac{1-(0.1)^n}{1-0.1}\right\}$$

$$\therefore t_n = \frac{0.5}{0.9}\{1-(0.1)^n\}$$

$$\therefore t_n = \frac{5}{9}\{1-(0.1)^n\}$$

Exercise 4.2 | Q 7.2 | Page 55

Find the n^{th} terms of the sequences: 0.2, 0.22, 0.222, …

SOLUTION

Let t_1 = 0.2, t_2 = 0.22, t_3 = 0.222 and so on.
t_1 = 0.2
t_2 = 0.22 = 0.2 + 0.02
t_3 = 0.222 = 0.2 + 0.02 + 0.002
∴ t_n = 0.2 + 0.02 + 0.002 + ... upto n terms
But 0.2, 0.02, 0.002, … upto n terms are in
G.P. with a = 0.2 and r = 0.1
∴ t_n = the sum of first n terms of the G.P.

$\therefore t_n = 0.2\left\{\frac{1-(0.1)^n}{1-0.1}\right\}$

$\therefore t_n = \frac{0.2}{0.9}\{1-(0.1)^n\}$

$\therefore t_n = \frac{2}{9}\{1-(0.1)^n\}$

Exercise 4.2 | Q 8 | Page 55

For a sequence, if $S_n = 2(3^n - 1)$, find the n^{th} term, hence show that the sequence is a G.P.

SOLUTION

$S_n = 2(3^n - 1)$
$\therefore S_{n-1} = 2(3^n - 1 - 1)$
But $t_n = S_n - S_{n-1}$
$= 2(3^n - 1) - 2(3^{n-1} - 1)$
$= 2(3^n - 1 - 3^{n-1} + 1)$
$= 2(3^n - 3^{n-1})$
$= 2(3^{n-1+1} - 3^{n-1})$
$\therefore tn = 2.3^{n-1}(3-1) = 4.3^{n-1}$

$\therefore t_{n+1} = 4.3^{(n+1-1)}$

$= 4(3)^n$

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

$\therefore \frac{t_{n+1}}{t_n} = \frac{4(3)^n}{4(3)^{n-1}}$

$= 3$

= constant, for all $n \in N$

$\therefore$ the sequence is a G.P. with $t_n = 4(3)^{n-1}$.

Exercise 4.2 | Q 9 | Page 55

If S, P, R are the sum, product and sum of the reciprocals of n terms of a G.P. respectively, then verify that $\left(\frac{S}{R}\right)^n = P^2$.

SOLUTION

Let a be the 1^{st} term and r be the common ratio of the G.P.

$\therefore$ the G.P. is a, ar, ar^2, ar^3, ..., ar^{n-1}

$$\therefore S = a + ar + ar^2 + \dots + ar^{n-1} = a\left(\frac{r^n - 1}{r - 1}\right)$$

$P = a(ar)(ar)^2 \dots (ar^{n-1})$

$= a^n \cdot r^{1+2+3+\dots+(n-1)}$

$= a^n \cdot r^{\frac{n(n-1)}{2}}$

$\therefore P = a^{2n} \cdot r^{n(n-1)}$ ...(i)

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \dots + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \dots + r^2 + r + 1}{a \cdot r^{n-1}}$$

$$= \frac{1 + r + r^2 + \dots + r^{n-2} + r^{n-1}}{a \cdot r^{n-1}}$$

1, r, r^2, ..., r^{n-1} are in G.P., with a = 1, r = r

$$\therefore R = \frac{1}{ar^{n-1}}\left(\frac{r^n - 1}{r - 1}\right) = \frac{1}{a^2 \cdot r^{n-1}} \times a \times \left(\frac{r^n - 1}{r - 1}\right)$$

$$\therefore R = \frac{1}{a^2 \cdot r^{n-1}} S$$

$$\therefore a^2 \cdot r^{n-1} = \frac{S}{R}$$

$$\therefore \left(a^2 \cdot r^{n-1}\right)^n = \left(\frac{S}{R}\right)^n$$

$$\therefore a^{2n} \cdot r^{(n-1)} = \left(\frac{S}{R}\right)^n$$

$$\therefore P^2 = \left(\frac{S}{R}\right)^n. \qquad \text{...[From (i)]}$$

Exercise 4.2 | Q 10 | Page 55

If S_n, S_{2n}, S_{3n} are the sum of n, 2n, 3n terms of a G.P. respectively, then verify that $S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.

SOLUTION

Let a and r be the 1st term and common ratio of the G.P. respectively.

$$\therefore S_n = a\left(\frac{r^n - 1}{r - 1}\right), S_{2n} = a\left(\frac{r^{2n} - 1}{r - 1}\right), S_{3n} = a\left(\frac{r^{3n} - 1}{r - 1}\right)$$

$$\therefore S_{2n} - S_n = a\left(\frac{r^{2n} - 1}{r - 1}\right) - a\left(\frac{r^n - 1}{r - 1}\right)$$

$$= \frac{a}{r - 1}\left(r^{2n} - 1 - r^n + 1\right)$$

$$= \frac{a}{r - 1}\left(r^{2n} - r^n\right)$$

$$= \frac{ar^n}{r - 1}\left(r^n - 1\right)$$

$$\therefore S_{2n} - S_n = \frac{r^n \cdot a(r^n - 1)}{r - 1} \qquad \text{....(i)}$$

$$S_{3n} - S_{2n} = a\left(\frac{r^{3n} - 1}{r - 1}\right) - a\left(\frac{r^{2n} - 1}{r - 1}\right)$$

$$= \frac{a}{r - 1}\left(r^{3n} - 1 - r^{2n} + 1\right)$$

$$= \frac{a}{r - 1}\left(r^{3n} - r^{2n}\right)$$

$= \frac{a}{r-1} \cdot r^{2n}(r^n - 1)$

$= a \cdot \left(\frac{r^n - 1}{r - 1}\right) \cdot r^{2n}$

$\therefore S_n(S_{3n} - S_{2n}) = \left[a \cdot \left(\frac{r^n - 1}{r - 1}\right)\right]\left[a \cdot \left(\frac{r^n - 1}{r - 1}\right) r^{2n}\right]$

$= \left[r^n \cdot \frac{a(r^n - 1)}{r - 1}\right]^2$

$\therefore S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$ ....[From (i)]

EXERCISE 4.3 [PAGES 56 - 57]

Exercise 4.3 | Q 1.1 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

SOLUTION

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

Here, $a = \frac{1}{2}, r = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$

Since, $|r| = \left|\frac{1}{2}\right| < 1$

$\therefore$ Sum to infinity exists.

Sum to infinity = $\frac{a}{1-r}$

$= \frac{\frac{1}{2}}{1 - \frac{1}{2}}$

= 1.

Exercise 4.3 | Q 1.2 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

SOLUTION

$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

a = 2, r = $\frac{\frac{4}{3}}{2} = \frac{2}{3}$

Since, | r | = $\left|\frac{2}{3}\right| < 1$

∴ Sum to infinity exists.

Sum to infinity = $\frac{a}{1-r}$

= $\frac{2}{1-\frac{2}{3}}$

= 6.

Exercise 4.3 | Q 1.3 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

$-3, 1, \frac{-1}{3}, \frac{1}{9}, \dots$

SOLUTION

$-3, 1, \frac{-1}{3}, \frac{1}{9}, \dots$

a = − 3, r = $-\frac{1}{3}$

Since, | r | = $\left|-\frac{1}{3}\right| < 1$

∴ Sum to infinity exists.

Sum to infinity = $\frac{a}{1-r}$

$= \frac{-3}{1-\left(-\frac{1}{3}\right)}$

$= -\frac{9}{4}.$

Exercise 4.3 | Q 1.4 | Page 57

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it

$\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$

SOLUTION

$\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \ldots$

$a = \frac{1}{5}, r = \frac{\frac{-2}{5}}{\frac{1}{5}} = -2$

Since, | r | = |– 2| > 1

∴ Sum to infinity does not exist.

Exercise 4.3 | Q 2.1 | Page 57

Express the following recurring decimals as a rational number: $0.\overline{32}$

SOLUTION

$0.\overline{32}$ = 0.323232...

= 0.32 + 0.0032 + 0.000032 + ...

Here, 0.32, 0.0032, 0.000032, ... are in G.P.

with a = 0.32 and r = 0.01

Since, | r | = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

∴ $0.\overline{32} = \frac{0.32}{1-(0.01)} = \frac{0.32}{0.99}$

∴ $0.\overline{32}$ = 32/99.

Exercise 4.3 | Q 2.2 | Page 57

Express the following recurring decimals as a rational number : 3.5

SOLUTION

$3.\dot{5}$ = 3.555... = 3 + 0.5 + 0.05 + 0.005 + ...

Here, 0.5, 0.05, 0.005, ... are in G.P. with

a = 0.5 and r = 0.1.

Since, | r | = |0.1| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

= $\frac{0.5}{1-(0.1)}$

= $\frac{0.5}{0.9}$

= $\frac{5}{9}$

∴ $3.\dot{5} = 3 + \frac{5}{9}$

= $\frac{32}{9}$.

Exercise 4.3 | Q 2.3 | Page 57

Express the following recurring decimals as a rational number: $4.\overline{18}$

SOLUTION

$4.\overline{18}$ = 4.181818...

= 4 + 0.18 + 0.0018 + 0.000018 + ...

Here, 0.18, 0.0018, 0.000018, ... are in G.P.

with a = 0.18 and r = 0.01

Since, | r | = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

= $\frac{0.18}{1-(0.01)}$

= $\frac{0.18}{0.99}$

= $\frac{18}{19}$

= $\frac{2}{11}$

∴ $4.\overline{18}$ = $4+\frac{2}{11}$

= $\frac{46}{11}$.

Exercise 4.3 | Q 2.4 | Page 57

Express the following recurring decimals as a rational number: $0.3\overline{45}$

SOLUTION

$0.3\overline{45}$ = 0.3454545...

= 0.3 + 0.045 + 0.00045 + 0.0000045 + ...

Here, 0.045, 0.00045, 0.0000045, ... are in

G.P. with a = 0.045, r = 0.01

Since, | r | = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

= $\frac{0.045}{1-0.01}$

= $\frac{0.045}{0.99}$

= $\frac{45}{990}$

∴ $0.3\overline{45} = 0.3 + \frac{45}{990}$

= $\frac{3}{10} + \frac{1}{22}$

= $\frac{33+5}{110}$

= $\frac{38}{110}$

= $\frac{19}{55}$

Alternate Method:

$0.3\overline{45} = \frac{0.3\overline{45}}{10}$

= $\frac{3 + 0.45 + 0.0045 + 0.000045 + ...}{10}$

Here, 0.45, 0.0045, 0.000045... are in G.P.

with a = 0.45 and r = 0.01

Since, | r | = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\frac{a}{1-r}$

$= \dfrac{0.45}{1 - 0.01}$

$= \dfrac{0.45}{0.99}$

$= \dfrac{45}{99}$

$= \dfrac{5}{11}$

$\therefore 0.3\overline{45} = \dfrac{3 + \frac{5}{11}}{10}$

$= \dfrac{\frac{38}{11}}{10}$

$= \dfrac{19}{55}$.

Exercise 4.3 | Q 2.5 | Page 57

Express the following recurring decimals as a rational number: $3.4\overline{56}$

SOLUTION

$3.4\overline{56}$ = 3.4565656 ...

= 3.4 + 0.056 + 0.00056 + 0.0000056 + ...

Here, 0.056, 0.00056, 0.0000056, ... are in

G.P. with a = 0.056 and r = 0.01

Since, | r| = |0.01| < 1

∴ Sum to infinity exists.

∴ Sum to infinity = $\dfrac{a}{1 - r}$

$= \dfrac{0.056}{1 - 0.01}$

$= \frac{0.056}{0.99}$

$= \frac{56}{990}$

$\therefore 3.4\overline{56} = 3.4 + \frac{56}{990}$

$= \frac{34}{10} + \frac{56}{990}$

$= \frac{3366 + 56}{990}$

$= \frac{3422}{990}$

$= \frac{1711}{495}$.

Exercise 4.3 | Q 3 | Page 57

If the common ratio of a G.P. is 2/3 and sum of its terms to infinity is 12. Find the first term.

SOLUTION

$r = \frac{2}{3}$ sum to infinity = 12 ...[Given]

Sum to infinity = $\frac{a}{1 - r}$

$\therefore 12 = \frac{a}{1 - \frac{2}{3}}$

$\therefore a = 12 \times \frac{1}{3}$

$\therefore a = 4$.

Exercise 4.3 | Q 4 | Page 57

If the first term of a G.P. is 16 and sum of its terms to infinity is 176/5, find the common ratio.

SOLUTION

a = 16, sum to infinity = $\frac{176}{5}$ ...[Given]

Sum to infinity = $\frac{a}{1-r}$

$\therefore \frac{176}{5} = \frac{16}{1-r}$

$\therefore \frac{11}{5} = \frac{1}{1-r}$

$\therefore 11 - 11r = 5$

$\therefore 11r = 6$

$\therefore r = \frac{6}{11}$.

Exercise 4.3 | Q 5 | Page 57

The sum of the terms of an infinite G.P. is 5 and the sum of the squares of those terms is 15. Find the G.P.

SOLUTION

Let the required G.P. be a, ar, ar^2, ar^3, ...

Sum to infinity of this G.P. = 5

$\therefore 5 = \frac{a}{1-r}$

$\therefore a = 5(1 - r)$ ...(i)

Also, the sum of the squares of the terms is 15.

$\therefore (a^2 + a^2r^2 + a^2r^4 + ...) = 15$

$\therefore 15 = \frac{a^2}{1-r^2}$

$\therefore 15(1 - r^2) = a^2$

$\therefore 15(1 - r)(1 + r) = 25(1 - r)^2$ ...[From (i)]

$\therefore$ 3 (1 + r) = 5(1 – r)

$\therefore$ 3 + 3r = 5 – 5r

$\therefore$ 8r = 2

$\therefore r = \frac{1}{4}$

$\therefore a = 5\left(1 - \frac{1}{4}\right) = 5\left(\frac{3}{4}\right) = \frac{15}{4}$

$\therefore$ Required G.P. is a, ar, ar^2, ar^3, ...

i.e., $\frac{15}{4}, \frac{15}{16}, \frac{15}{64}, \ldots$

Exercise 4.4 | Q 1.1 | Page 60

Verify whether the following sequences are H.P. : $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$

SOLUTION

$\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots$

Here, the reciprocal sequence is 3, 5, 7, 9, ...

$\therefore t_1 = 3, t_2 = 5, t_3 = 7, \ldots$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant

$\therefore$ The reciprocal sequence is an A.P.

$\therefore$ the given sequence is H.P.

Exercise 4.4 | Q 1.2 | Page 60

Verify whether the following sequences are H.P. : $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

SOLUTION

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \ldots$

Here, the reciprocal sequence is 3, 6, 9, 12 ...

$\therefore t_1 = 3, t_2 = 6, t_3 = 9, t_4 = 12, \ldots$

$\because t_2 - t_1 = 6, t_3 = 9, t_4 = 12, \ldots$

∴ The reciprocal sequence is an A.P.

∴ The given sequence is H.P.

Exercise 4.4 | Q 1.3 | Page 60

Verify whether the following sequences are H.P. : $\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \ldots$

SOLUTION

$\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \ldots$

Here, the reciprocal sequence is

$\therefore t_1 = 7, t_2 = 9, t_3 = 11, t_4 = 13, \ldots$

$\because t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant

∴ The reciprocal sequence is an A.P.

∴ The given sequence is H.P.

Exercise 4.4 | Q 2.1 | Page 60

Find the n^{th} term and hence find the 8^{th} term of the following H.P.s: $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$

SOLUTION

$\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \ldots$ are in H.P.

∴ 2, 5, 8, 11, ... are in A.P.

∴ a = 2, d = 3

tn = a + (n – 1)d

= 2 + (n – 1)(3)

= 3n – 1

$\therefore$ n^{th} term of H.P. is $\frac{1}{3n-1}$

$\therefore$ 8^{th} term of H.P. = $\frac{1}{3(8)-1} = \frac{1}{23}$

Exercise 4.4 | Q 2.2 | Page 60

Find the n^{th} term and hence find the 8^{th} term of the following H.P.s: $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...$

SOLUTION

$\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...$ are in H.P.

$\therefore$ 4, 6, 8, 10, ... are in A.P.

$\therefore$ a = 4, d = 2

tn = a + (n – 1)d

= 4 + (n – 1) (2)

= 2n + 2

$\therefore$ n^{th} term of H.P. = $\frac{1}{2n+2}$

$\therefore$ 8^{th} term of H.P. = $\frac{1}{2(8)+2} = \frac{1}{18}$

Exercise 4.4 | Q 2.3 | Page 60

Find the n^{th} term and hence find the 8^{th} term of the following H.P.s: $\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, ...$

SOLUTION

$\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, ...$ are in H.P.

5, 10, 15, 20, ... are in H.P.

$\therefore$ a = 5, d = 5

t_n = a + (n – 1)d

$= 5 + (n - 1)(5)$

$= 5n$

$\therefore$ n^{th} term of H.P. $= \dfrac{1}{5n}$

$\therefore$ 8^{th} term of H.P. $= \dfrac{1}{5(8)}$

$= \dfrac{1}{40}$.

Exercise 4.4 | Q 3 | Page 60

Find A.M. of two positive numbers whose G.M. and H.M. are 4 and 16/5.

SOLUTION

G.M. = 4, H.M. = $\dfrac{16}{5}$

$\because$ $(G.M.)^2 = (A.M.)\,(H.M.)$

$\therefore 16 = \mathbf{A.M} \times \dfrac{16}{5}$

$\therefore$ A.M. = 5

Exercise 4.4 | Q 4 | Page 60

Find H.M. of two positive numbers whose A.M. and G.M. are 15/2 and 6.

SOLUTION

A.M. = $\dfrac{15}{2}$, G.M. = 6

Now, $(G.M.)^2 = (A.M.)\,(H.M.)$

$\therefore 6^2 = \dfrac{15}{2} \times \mathbf{H.M}$

$\therefore$ H.M. = $36 \times \frac{2}{15}$

$\therefore$ H.M. = $\frac{24}{5}$

Exercise 4.4 | Q 5 | Page 60

Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48.

SOLUTION

A.M. = 75, H.M. = 48
$\because$ $(G.M.)^2$ = (A.M.) (H.M.)
$\therefore$ $(G.M.)^2$ = 75 x 48
= 25 x 3 x 16 x 3
= $5^2 \times 4^2 \times 3^2$
$\therefore$ G.M. = 5 x 4 x 3
$\therefore$ G.M. = 60

Exercise 4.4 | Q 6 | Page 60

Insert two numbers between 1/7 and 1/13 so that the resulting sequence is a H.P.

SOLUTION

Let the required numbers be $\frac{1}{H_1}$ and $\frac{1}{H_2}$.

$\therefore \frac{1}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{13}$ are in H.P.

$\therefore$ 7, H_1, H_2 and 13 are in A.P.

$\therefore t_1 = a = 7$ and $t_4 = a + 3d = 13$

$\therefore 7 + 3d = 13$

$\therefore 3d = 6$

$\therefore d = 2$

$\therefore H_1 = t_2 = a + d = 7 + 2 = 9$

and $H_2 = t_3 = a + 2d = 7 + 2(2) = 11$

$\therefore$ 1/9 and 1/11 are the required numbers to be inserted between 1/7 and 1/13 so that the resulting sequence is a H.P.

Exercise 4.4 | Q 7 | Page 60

Insert two numbers between 1 and –– 27 so that the resulting sequence is a G.P.

SOLUTION

Let the required numbers be G_1 and G_2.
∴ 1, G_1, G_2, – 27 are in G.P.
∴ $t_1 = 1$, $t_2 = G_1$, $t_3 = G_2$, $t_4 = -27$
∴ $t_1 = a = 1$
$t_n = ar^{n-1}$
∴ $t_4 = (1)^{4-1}$
∴ $-27 = r^3$
∴ $r^3 = (-3)^3$
∴ $r = -3$
∴ $G_1 = t_2 = ar = 1(-3) = -3$
$G_2 = t_3 = ar^2 = 1(-3)^2 = 9$
∴ – 3 and 9 are the required numbers to be inserted between 1 and –– 27 so that th resulting sequence is a G.P.

Exercise 4.4 | Q 8 | Page 60

Find two numbers whose A.M. exceeds their G.M. by 1/2 and their H.M. by 25/26.

SOLUTION

Let a, b be the two numbers.

$$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$$

According to the given conditions,

$$A = G + \frac{1}{2}, A = H + \frac{25}{26}$$

$$\therefore G = A - \frac{1}{2}, H = A - \frac{25}{26} \quad \text{...(i)}$$

Now, $G^2 = AH$

$$\left(A - \frac{1}{2}\right)^2 = A\left(A - \frac{25}{26}\right)$$

$$\therefore A^2 - A + \frac{1}{4} = A^2 - \frac{25}{26}A$$

$$\therefore A - \frac{25}{26}A = \frac{1}{4}$$

$$\therefore \frac{1}{26}A = \frac{1}{4}$$

$$\therefore A = \frac{13}{2} \qquad \text{...(ii)}$$

$\therefore$ G = 6 ...[From (i) and (ii)]

$$\therefore \frac{a+b}{2} = \frac{13}{2} \text{ and } \sqrt{ab} = 6$$

$\therefore$ a + b = 13,

$\therefore$ b = 13 – a ...(iii)

and ab = 36

$\therefore$ a(13 – a) = 36 ...[From (iii)]

$\therefore a^2 - 13a + 36 = 0$

$\therefore$ (a – 4)(a – 9) = 0

$\therefore$ a = 4 or a = 9

When a = 4, b = 13 – 4 = 9

When a = 9, b = 13 – 9 = 4

$\therefore$ the two numbers are 4 and 9.

Exercise 4.4 | Q 9 | Page 61

Find two numbers whose A.M. exceeds G.M. by 7 and their H.M. by 63/5.

SOLUTION

Let a, b be the two numbers.

$$A = \frac{a+b}{2}, G = \sqrt{ab}, H = \frac{2ab}{a+b}$$

According to the given conditions,

$A = G + 7, A = H + \frac{63}{5}$

$\therefore G = A - 7,$ ...(i)

$H = A - \frac{63}{5}$

Now, $G^2 = AH$

$\therefore (A - 7)^2 = A\left(A - \frac{63}{5}\right)$

$\therefore A^2 - 14A + 49 = A^2 - \frac{63A}{5}$

$\therefore 14A - \frac{63A}{5} = 49$

$\therefore \frac{7A}{5} = 49$

$\therefore A = 35$

$\therefore \frac{a + b}{2} = 35$

$\therefore a + b = 70$

$\therefore b = 70 - a$ ...(ii)

$G = A - 7$ ...[From (i)]

$= 35 - 7$

$\therefore G = 28$

$\therefore \sqrt{ab} = 28$

$\therefore ab = 28^2 = 784$

$\therefore a(70 - a) = 784$ ...[From (ii)]

$\therefore 70a - a^2 = 784$

$\therefore a^2 - 70a + 784$

$\therefore a^2 - 56a - 14a + 784 = 0$

∴ (a – 56) (a – 14) = 0

∴ a = 14 or a = 56

When a = 14, b = 70 – 14 = 14

When a = 56, b = 70 – 56 = 14

∴ the two numbers are 14 and 56.

EXERCISE 4.5 [PAGE 63]

Exercise 4.5 | Q 1 | Page 63

Find the sum $\sum_{r=1}^{n}(r+1)(2r-1)$.

SOLUTION

$$\sum_{r=1}^{n}(r+1)(2r-1)$$

$$= \sum_{r=1}^{n}\left(2r^2+r-1\right)$$

$$= 2\sum_{r=1}^{n}r^2+\sum_{r=1}^{n}r-\sum_{r=1}^{n}1$$

$$= 2.\frac{n(n+1)(2n+1)}{6}+\frac{n(n+1)}{2}-n$$

$$= \frac{n}{6}\left[2\left(2n^2+3n+1\right)+3(n+1)-6\right]$$

$$= \frac{n}{6}\left(4n^2+6n+2+3n+3-6\right)$$

$$= \frac{n}{6}\left(4n^2+9n-1\right).$$

Exercise 4.5 | Q 2 | Page 63

Find $\sum_{r=1}^{n}\left(3r^2-2r+1\right)$.

SOLUTION

$$\sum_{r=1}^{n}\left(3r^2 - 2r + 1\right)$$

$$= 3\sum_{r=1} r^2 - 2\sum_{r=1}^{n} r + \sum_{r=1} 1$$

$$= 3.\ \frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + n$$

$$= \frac{n}{2}\left[2n^2 + 3n + 1\right) - 2(n+1) + 2\right]$$

$$= \frac{n}{2}\left(2n^2 + 3n + 1 - 2n - 2 + 2\right)$$

$$= \frac{n}{2}\left(2n^2 + n + 1\right).$$

Exercise 4.5 | Q 3 | Page 63

Find $\sum_{r-1}^{n} \frac{1+2+3+...+r}{r}$.

SOLUTION

$$\sum_{r=1}^{n}\left(\frac{1+2+3+...+r}{r}\right)$$

$$= \sum_{r=1}^{n} r\frac{(r+1)}{2r}$$

$$= \frac{1}{2}\sum_{r=1}^{n}(r+1)$$

$$= \frac{1}{2}\left[\sum_{r=1}^{n} + \sum_{r=1}^{n} 1\right]$$

$= \frac{1}{2}\left[\frac{n(n+1)}{2} + n\right]$

$= \frac{n}{4}[(n+1) + 2]$

$= \frac{n}{4}(n+3).$

Exercise 4.5 | Q 4 | Page 63

Find $\sum_{r=1}^{n} \frac{1^3 + 2^3 + ... + r^3}{r(r+1)}$.

SOLUTION

We know that,

$1^3 + 2^3 + 3^3 + ... + n^3 = \frac{n^2(n+1)^2}{4}$

$\therefore 1^3 + 2^3 + 3^3 + ... + r^3 = \frac{r^2(r+1)^2}{4}$

$\therefore \frac{1^3 + 2^3 + 3^3 + ... + r^3}{r(r+1)} = \frac{r(r+1)}{4}$

$\therefore \sum_{r=1}^{n}\left[\frac{1^3 + 2^3 + 3^3 + ... + r^3}{r(r+1)}\right]$

$= \sum_{r=1}^{n} \frac{r(r+1)}{4}$

$= \frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$

$= \frac{1}{4} \cdot \frac{n(n+1)}{2}\left(\frac{2n+1}{3} + 1\right)$

$= \frac{n(n+1)}{8}\left(\frac{2n+1+3}{3}\right)$

$$= \frac{n(n+1)(2n+4)}{24}$$

$$= \frac{2n(n+1)(n+2)}{24}$$

$$= \frac{n(n+1)(n+2)}{12}.$$

Exercise 4.5 | Q 5 | Page 63

Find the sum 5 x 7 + 7 x 9 + 9 x 11 x 13 + ... upto n terms.

SOLUTION

5 x 7 + 7 x 9 + 9 x 11 x 13 + ... upto n terms
Now, 5, 7, 9, 11, … are in A.P.
r^{th} term = 5 + (r – 1) (2) = 2r + 3
7, 9, 11, . … are in A.P.
r^{th} term = 7 + (r – 1) (2) = 2r + 5
∴ 5 x 7 + 7 x 9 + 9 x 11 x 13 + ... upto n terms

$$= \sum_{r-1}^{n}(2r+3)(2r+5)$$

$$= \sum_{r-1}^{n}\left(4r^2+16r+15\right)$$

$$= 4\sum_{r-1}^{n} r^2 + 6\sum_{r-1}^{n} r + 15\sum_{r-1}^{n} 1$$

$$= 4\frac{n(n+1)(2n+1)}{6} + 16\frac{n(n+1)}{2} + 15n$$

$$= \frac{n}{3}\left[2\left(2n^2+3n+1\right)+24(n+1)+45\right]$$

$$= \frac{n}{3}\left(4n^2+6n+2+24n+24+45\right)$$

$$= \frac{n}{3}\left(4n^2+30n+71\right).$$

Exercise 4.5 | Q 6 | Page 63

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + \ldots$ upto n terms.

SOLUTION

$2^2 + 4^2 + 6^2 + 8^2 + \dots$ upto n terms

$= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 + \times 4)^2 + \dots$

$= \sum_{r=1}^{n} (2r)^2$

$= 4\sum_{r=1}^{n} r^2$

$= \dfrac{4.\,n(n+1)(2n+1)}{6}$

$= \dfrac{2n(n+1)(2n+1)}{3}$.

Exercise 4.5 | Q 6 | Page 63

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + \dots$ upto n terms.

SOLUTION

$2^2 + 4^2 + 6^2 + 8^2 + \dots$ upto n terms

$= (2 \times 1)^2 + (2 \times 2)^2 + (2 \times 3)^2 + (2 + \times 4)^2 + \dots$

$= \sum_{r=1}^{n} (2r)^2$

$= 4\sum_{r=1}^{n} r^2$

$= \dfrac{4.\,n(n+1)(2n+1)}{6}$

$= \dfrac{2n(n+1)(2n+1)}{3}$.

Exercise 4.5 | Q 7 | Page 63

Find $(70^2 - 69^2) + (68^2 - 67^2) + \dots + (2^2 - 1^2)$

SOLUTION

Let S = $(70^2 - 69^2) + (68^2 - 67^2) + \dots + (2^2 - 1^2)$

$\therefore$ S = $(2^2 - 1^2) + (4^2 - 3^2) + \dots + (70^2 - 69^2)$
Here, 2, 4, 6, …, 70 is an A.P. with rth term = 2r
and 1, 3, 5, …, 69 in A.P. with rth term = 2r – 1

$$\therefore S = \sum_{r=1}^{35}\left[(2r)^2 - (2r-1)^2\right]$$

$$= \sum_{r=1}^{35}\left[4r^2 - \left(4r^2 - 4r + 1\right)\right]$$

$$= \sum_{r=1}^{35}(4r - 1)$$

$$= 4\sum_{r-1}^{35} r - \sum_{r=1}^{35} 1$$

$$= 4.\frac{35 \times 36}{2} - 35$$

$$= (72 - 1)(35)$$

$$= 71 \times 35$$

$$= 2485.$$

Exercise 4.5 | Q 8 | Page 63
Find the sum 1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... + (2n – 1) (2n + 1) (2n + 3)

SOLUTION

1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... + (2n – 1) (2n + 1) (2n + 3)
Now, 1, 3, 5, 7, … are in A.P. with a = 1 and d = 2.
$\therefore$ r^{th} term = 1 + (r – 1)2 = 2r – 1
3, 5, 7, 9, … are in A.P. with a = 3 and d = 2
$\therefore$ r^{th} term = 3 + (r – 1)2 = 2r + 1
and 5, 7, 9, 11, … are in A.P. with a = 5 and d = 2.
$\therefore$ r^{th} term = 5 +(r – 1)2 = 2r + 3
$\therefore$ 1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... upto n terms

$= \sum_{r=1}^{n} (2r-1)(2r+1)(2r+3)$

$= \sum_{r=1}^{n} \left(4r^2-1\right)(2r+3)$

$= \sum_{r=1}^{n} \left(8r^3+12r^2-2r-3\right)$

$= 8\sum_{r=1}^{n} r^3 + 12\sum_{r=1}^{n} r^2 - 2\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$

$= 8\left\{\frac{n(n+1)}{2}\right\}^2 + 12\left\{\frac{n(n+1)(2n+1)}{6}\right\} - 2\left\{\frac{n(n+1)}{2}\right\} - 3n$

$= 2n^2(n + 1)^2 + 2n(n + 1)(2n + 1) - n(n + 1) - 3n$

$= n(n + 1)[2n(n + 1) + 4n + 2 - 1] - 3n$

$= n(n + 1)(2n^2 + 6n + 1) - 3n$

$= n(2n^3 + 8n^2 + 7n + 1 - 3)$

$= n(2n^3 + 8n^2 + 7n - 2)$.

Exercise 4.5 | Q 9 | Page 63

Find n, if $\frac{1\times 2+2\times 3+3\times 4+4\times 5+\text{...upto n terms}}{1+2+3+4+\text{...upto n terms}} = \frac{100}{3}$.

SOLUTION

$$\frac{1\times 2+2\times 3+3\times 4+4\times 5+\text{...upto n terms}}{1+2+3+4+\text{...upto n terms}} = \frac{100}{3}$$

$\therefore \frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r} = \frac{100}{3}$

$\therefore \frac{\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r}{\sum_{r=1}^{n} r} = \frac{100}{3}$

$$\therefore \frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n(n+1)}{2}} = \frac{100}{3}$$

$$\therefore \frac{\frac{n(n+1)}{6}\left[(2n+1)+3\right]}{\frac{n(n+1)}{2}} = \frac{100}{3}$$

$$\therefore \frac{2(n+2)}{3} = \frac{100}{3}$$

$\therefore$ n + 2 = 50

$\therefore$ n = 48.

Exercise 4.5 | Q 10 | Page 63

If S_1, S_2 and S_3 are the sums of first n natural numbers, their squares and their cubes respectively, then show that: $9S_2^2 = S_3(1 + 8S_1)$.

SOLUTION

$S_1 = 1 + 2 + 3 + ... + n = \sum_{r=1}^{n} = \frac{n(n+1)}{2}$

$S_2 = 1^2 + 2^2 + 3^2 + ... n^2 = \frac{n(n+1)(2n+1)}{6}$

$S_3 = 1^3 + 2^3 + 3^3 + ... + n3 = \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

R.H.S. = $S_3(1 + 8S_1)$

$= \frac{n^2(n+1)^2}{4}\left[1 + 8 \cdot \frac{n(n+1)}{4}\right]$

$= \frac{n^2(n+1)^2}{4}\left(1 + 4n^2 + 4n\right)$

$$= \frac{n^2(n+1)^2}{4}(2n+1)^2$$

$$= \frac{9.\,n^2(n+1)^2(2n+1)^2}{36}$$

MISCELLANEOUS EXERCISE 4 [PAGES 63 - 64]

Miscellaneous Exercise 4 | Q 1 | Page 63

In a G.P., the fourth term is 48 and the eighth term is 768. Find the tenth term.

SOLUTION

Given, $t_4 = 48$, $t_8 = 768$
$t_n = ar^{n-1}$
$\therefore t_4 = ar^3$
$\therefore ar^3 = 48$ ...(i)
and $ar^7 = 768$ ...(ii)
Equation (ii) ÷ equation (i), we get

$$\frac{ar^7}{ar^3} = \frac{768}{48}$$

$\therefore r^4 = 16$
$\therefore r = 2$
Substituting r = 2 in (i), we get
$a.(2^3) = 48$
$\therefore a = 6$
$\therefore t_{10} = ar^9$
$\therefore t_{10} = ar^9$
$= 6(2^9)$
= 3072.

Miscellaneous Exercise 4 | Q 2 | Page 63

For a G.P. $a = \frac{4}{3}$ and $t_7 = \frac{243}{1024}$, find the value of r.

SOLUTION

Given, $a = \frac{4}{3}$ and $t_7 = \frac{243}{1024}$

$t_n = ar^{n-1}$

$\therefore t_7 = ar^6$

$\therefore \frac{243}{1024} = \frac{4}{3}r^6$

$\therefore r^6 = \frac{3^6}{4^6}$

$\therefore r = \frac{3}{4}$

Miscellaneous Exercise 4 | Q 3 | Page 64

For a sequence, if tn = $\frac{5^{n-2}}{7^{n-3}}$, verify whether the sequence is a G.P. If it is a G.P., find its first term and the common ratio.

SOLUTION

The sequence (t_n) is a G.P. if

$\frac{t_{n+1}}{t_n}$ = constant for all $n \in N$.

Now, $t_n = \frac{5^{n-2}}{7^{n-3}}$

$\therefore t_{n+1} = \frac{5^{n+1-2}}{7^{n+1-3}} = \frac{5^{n-1}}{7^{n-2}}$

$\therefore \frac{t_{n+1}}{t_n} = \frac{5^{n-1}}{7^{n-2}} = \frac{7^{n-3}}{5^{n-2}}$

$= 5^{(n-1)-(n-2)} \times 7^{(n-3)-(n-2)}$

$= 5^{(1)} \times 7^{-1}$

$= \frac{5}{7}$

= constant , for all n ∈ N.

∴ the sequence is a G.P. with common ratio (r) = $\frac{5}{7}$

and first term = t_1 = $\frac{5^{1-2}}{7^{1-3}}$

= $\frac{5^{-1}}{7^{-2}}$

= $\frac{7^2}{5}$

= $\frac{49}{5}$.

Miscellaneous Exercise 4 | Q 4 | Page 64

Find three numbers in G.P., such that their sum is 35 and their product is 1000.

SOLUTION

Let the three numbers in G.P. be $\frac{a}{r}$, a, ar.

According to the first condition,

$\frac{a}{r} + a + ar$ = 35

∴ $a\left(\frac{1}{r} + 1 + r\right)$ = 35 ...(i)

According to the second condition,

$\left(\frac{a}{r}\right)(a)(ar)$ = 1000

∴ a^3 = 1000

∴ a = 10

Substituting the value of a in (i), we get

$$10\left(\frac{1}{r}+1+r\right) = 35$$

$$\therefore \frac{1}{r}+r+1=\frac{35}{10}$$

$$\therefore \frac{1}{r}+r=\frac{35}{10}-1$$

$$\therefore \frac{1}{r}+r=\frac{25}{10}$$

$$\therefore \frac{1}{r}+r=\frac{5}{2}$$

$\therefore 2r^2 - 5r + 2 = 0$

$\therefore (2r - 1)(r - 2) = 0$

$$\therefore r = \frac{1}{2} \text{ or } r=2$$

When $r = \frac{1}{2}, a=10$

$$\frac{a}{r}=\frac{10}{\left(\frac{1}{2}\right)}=20, a=10 \text{ and } ar=10\left(\frac{1}{2}\right) = 5$$

When $r = 2$, $a = 10$

$$\frac{a}{r}=\frac{10}{2} = 5, a = 10 \text{ and } ar = 10(2) = 20$$

∴ the three numbers in G.P. are 20, 10, 5 or 5, 10, 20.

Miscellaneous Exercise 4 | Q 5 | Page 64

Find four numbers in G. P. such that sum of the middle two numbers is 10/3 and their product is 1.

SOLUTION

Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}, ar, ar^3$.

According to the second condition,

$\frac{a}{r^3}\left(\frac{a}{r}\right)(ar)\left(ar^3\right) = 1$

$\therefore a^4 = 1$

$\therefore a = 1$

According to the first condition,

$$\frac{a}{r}+ar\ \frac{10}{3}$$

$$\therefore \frac{1}{r}+(1)r=\frac{10}{3}$$

$$\therefore \frac{1+r^2}{r}=\frac{10}{3}$$

$\therefore 3 + 3r^2 = 10r$

$\therefore 3r^2 - 10r + 3 = 0$

$\therefore (r - 3)(3r - 1) = 0$

$\therefore r = 3$ or $r = \frac{1}{3}$

When = 3, a = 1

$\frac{a}{r^3}=\frac{1}{(3)^3}=\frac{1}{27}, \frac{a}{r}=\frac{1}{3}$, $ar = 1(3) = 3$ and $ar^3 = 1(3)^3 = 27$

When $r = \frac{1}{3}$, a = 1

$$\frac{a}{r^3}=\frac{1}{\left(\frac{1}{3}\right)^3}=27, \frac{a}{r}=\frac{1}{\left(\frac{1}{3}\right)}=3,$$

$$ar=1\left(\frac{1}{3}\right)=\frac{1}{3}\ \text{and}\ r^3=1\left(\frac{1}{3}\right)^3=\frac{1}{27}$$

$\therefore$ the four numbers in G.P.are

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Miscellaneous Exercise 4 | Q 6 | Page 64

Find five numbers in G.P. such that their product is 243 and sum of second and fourth number is 10.

SOLUTION

Let the five numbers in G.P. be

$\frac{a}{r^2}, \frac{a}{r}$, a, ar, ar^2.

According to the first condition,

$$\frac{a}{r^2} \times \frac{a}{r} \times a \times ar \times ar^2 = 243$$

$\therefore a^5 = 243$

$\therefore a = 3$

According to the second condition,

$$\frac{a}{r} + ar = 10$$

$$\therefore \frac{1}{r} + r = \frac{10}{a}$$

$$\therefore \frac{1 + r^2}{r} = \frac{10}{3}$$

$\therefore 3r^2 - 10r + 3 = 0$

$\therefore 3r^2 - 9r - r + 3 = 0$

$\therefore (3r - 1)(r - 3) = 0$

$$\therefore r = \frac{1}{3}, 3$$

When a = $3, r = \frac{1}{3}$

$$\frac{a}{r^2} = 27, \frac{a}{r} = 9, a = 3, ar = 1, ar^2 = \frac{1}{3}$$

When a = 3, r = 3

$$\frac{a}{r^2} = \frac{1}{3}, \frac{a}{r} = 1, a = 3, ar = 9, ar^2 = \frac{1}{3}$$

∴ the five numbers in G.P. are

27, 9, 3, 1, $\frac{1}{3}$ or $\frac{1}{3}$, 1, 3, 9, 27.

Miscellaneous Exercise 4 | Q 7 | Page 64

For a sequence $S_n = 4(7^n - 1)$, verify whether the sequence is a G.P.

SOLUTION

$S_n = 4(7n - 1)$
∴ $S_{n-1} = 4(7^{n-1} - 1)$
But, $t_n = S_n - S_{n-1}$
$= 4(7^n - 1) - 4(7^{n-1} - 1)$
$= 4(7^n - 1 - 7^{n-1} + 1)$
$= 4(7^{n-1+1} - 7^{n-1})$
$= 4.7^{n-1}(7 - 1)$
∴ $t_n = 27.7^{n-1}$
∴ $t_{n+1} = 24(7)^{n+1-1}$
$= 24(7)n$

The sequence (t_n) is a G.P., if $\frac{t_{n+1}}{t_n}$ = constant for all n ∈ N.

$$\therefore \frac{t_{n+1}}{t_n} = \frac{24(7)^n}{24(7)^{n-1}}$$

= 7

= constant, for all n ∈ N

∴ the sequence is a G.P.

Miscellaneous Exercise 4 | Q 8 | Page 64

Find 2 + 22 + 222 + 2222 + … upto n terms.

SOLUTION

S_n = 2 + 22 + 222 +... upto n terms

= 2(1 + 11 + 111+ ... upto n terms)

$= \frac{2}{9}$ (9 + 99 + 999 + upto n terms)

$= \frac{2}{9}$[(10 – 1) + (100 – 1) + (1000 – 1) + ... upto n terms]

$= \frac{2}{9}$[(10 – + 100 + 100 + ... upto n terms) – (1 + 1 + 1 ... n terms)]

Since, 10, 100,1000, ... n terms are in G.P. with a= 10, r = $\frac{100}{100}$ = 10

$\therefore S_n = \frac{2}{9}\left[10\left(\frac{10^n - 1}{10 - 1}\right) - n\right]$

$= \frac{2}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$

$\therefore S_n = \frac{2}{81}[10(10^n - 1) - 9n]$

Miscellaneous Exercise 4 | Q 9 | Page 64

Find the n^{th} term of the sequence 0.6, 0.66, 0.666, 0.6666, …

SOLUTION

0.6, 0.66, 0.666, 0.6666, …
∴ t_1 = 0.6
t_2 = 0.66 = 0.6 + 0.06
t_3 = 0.666 = 0.6 + 0.06 + 0.006
Hence, in general
t_n = 0.6 + 0.06 + 0.006 + …upto n terms.
The terms are in G.P. with

$a = 0.6,\ r = \dfrac{0.06}{0.6} = 0.1$

$\therefore t_n$ = the sum of first n terms of the G.P.

$\therefore\ t_n = 0.6\left[\dfrac{1-(0.1)^n}{1-0.1}\right]$

$= \dfrac{0.6}{0.9}[1-(0.1)^n]$

$\therefore\ t_n = \dfrac{6}{9}[1-(0.1)^n]$

$= \dfrac{2}{3}[1-(0.1)^n].$

Miscellaneous Exercise 4 | Q 10 | Page 64

Find $\sum_{r=1}^{n}(5r^2+4r-3)$.

SOLUTION

$\sum_{r=1}^{n}(5r^2+4r-3)$

$= 5\sum_{r=1}^{n} r^2 + 4\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1$

$= 5.\dfrac{n(n+1)(2n+1)}{6} + 4.\dfrac{n(n+1)}{2} - 3n$

$= \dfrac{n}{6}[5(2n^2+3n+1)+12(n+1)-18]$

$= \dfrac{n}{6}(10n^2+15n+5+12n+12-18)$

$= \dfrac{n}{6}(10n^2+27n-1).$

Miscellaneous Exercise 4 | Q 11 | Page 64

Find $\sum_{r=1}^{n} r(r-3)(r-2)$.

SOLUTION

$$\sum_{r=1}^{n} r(r-3)(r-2)$$

$$= \sum_{r=1}^{n} \left(r^3 - 5r^2 + 6r\right)$$

$$= \sum_{r=1}^{n} r^3 - 5\sum_{r=1}^{n} r^2 + 6\sum_{r=1}^{n} r$$

$$= \frac{n^2(n+1)^2}{4} - 5\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{12}[3n(n+1) - 10(2n+1) + 36]$$

$$= \frac{n(n+1)}{12}\left(3n^2 + 3n - 20n - 10 + 36\right)$$

$$= \frac{n(n+1)}{12}\left(3n^2 - 17n + 26\right).$$

Miscellaneous Exercise 4 | Q 12 | Page 64

Find $\sum_{r=1}^{n} \frac{1^2 + 2^2 + 3^2 + \ldots r^2}{2r+1}$.

SOLUTION

We know that

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\therefore 1^2 + 2^2 + 3^2 + \dots + r^2 = \frac{r(r+1)(2r+1)}{6}$$

$$\therefore \frac{1^2 + 2^2 + 3^2 + \dots r^2}{2r+1} = \frac{r(r-1)}{6}$$

$$\therefore \sum_{r=1}^{n} \left(\frac{1^2 + 2^2 + 3^2 + \dots r^2}{2r+1} \right)$$

$$= \sum_{r=1}^{n} \frac{r(r+1)}{6}$$

$$= \frac{1}{6} \sum_{r=1}^{n} (r^2 + r)$$

$$= \frac{1}{6} \left(\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r \right)$$

$$= \frac{1}{6} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

$$= \frac{1}{6} \times \frac{n(n+1)}{2} \left(\frac{2n+1}{3} + 1 \right)$$

$$= \frac{n(n+1)}{12} \left(\frac{2n+1+3}{3} \right)$$

$$= \frac{n(n+1)(2n+4)}{36}$$

$$= \frac{2n(n+1)(n+2)}{36}$$

$$= \frac{n(n+1)(n+2)}{18}$$

Miscellaneous Exercise 4 | Q 13 | Page 64

Find $\sum_{r=1}^{n} \frac{1^3 + 2^3 + 3^3 + \dots + r^3}{(r+1)^2}$

SOLUTION

$$\sum_{r=1}^{n} \frac{1^3 + 2^3 + 3^3 + \dots + r^3}{(r+1)^2}$$

$$= \sum_{r=1}^{n} \frac{r^2(r+1)^2}{4} \times \frac{1}{(r+1)^2}$$

$$= \frac{1}{4} \sum_{r=1}^{n} r^2$$

$$= \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)(2n+1)}{24}.$$

Miscellaneous Exercise 4 | Q 14 | Page 64

Find 2 x + 6 + 4 x 9 + 6 x 12 + ... upto n terms.

SOLUTION

2, 4, 6, ... are in A.P.

$\therefore$ r^{th} term = 2 + (r – 1) 2 = 2r

6, 9, 12, ... are in A.P.

$\therefore$ r^{th} term = 6 + (r – 1) (3) = (3r + 3)

$\therefore$ 2 x + 6 + 4 x 9 + 6 x 12 + ... upto n terms

$$= \sum_{r=1}^{n} 2r \times (3r+3)$$

$$= 6 \sum_{r=1}^{n} r^2 + 6 \sum_{r=1}^{n} r$$

$$= 6 \cdot \frac{n(n+1)(2n+1)}{6} + 6 \frac{n(n+1)}{2}$$

= n(n + 1) (2n + 1 + 3)

= 2n(n + 1)(n + 2).

Miscellaneous Exercise 4 | Q 15 | Page 64

Find $12^2 + 13^2 + 14^2 + 15^2 + \ldots + 20^2$.

SOLUTION

$12^2 + 13^2 + 14^2 + 15^2 + \ldots + 20^2$

$= (1^2 + 2^2 + 3^2 + 4^2 + \ldots + 20^2) - (1^2 + 2^2 + 3^2 + 4^2 + \ldots + 11^2)$

$$= \sum_{r=1}^{20} r^2 - \sum_{r=1}^{11} r^2$$

$$= \frac{20(20+1)(2\times 20+1)}{6} - \frac{11(11+1)(2\times 11+1)}{6}$$

$$= \frac{20\times 21\times 41}{6} - \frac{11\times 12\times 23}{6}$$

$= 2870 - 506$

$= 2364.$

Miscellaneous Exercise 4 | Q 16 | Page 64

Find $(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + .. + (2^2 - 1^2)$.

SOLUTION

$(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + .. + (2^2 - 1^2)$.

$= (50^2 + 48^2 + 46^2 + \ldots + 2^2) - (49^2 + 47^2 + 45^2 + \ldots + 1^2)$

$$= \sum_{r=1}^{25} (2r)^2 - \sum_{r=1}^{25} (2r-1)^2$$

$$= \sum_{r=1}^{254} 4r^2 - \sum_{r=1}^{25} \left(4r^2 - 4r + 1\right)$$

$$= \sum_{r=1}^{25} \left[4r^2 - \left(4r^2 - 4r + 1\right)\right]$$

$$= \sum_{r=1}^{25} (4r - 1)$$

$$= 4\sum_{r=1}^{25} r - \sum_{r=1}^{25} 1$$

$$= 4 \times \frac{25(25+1)}{2} - 25$$

$$= \frac{4(25)(26)}{2} - 25$$

= 1300 – 25

= 1275.

Miscellaneous Exercise 4 | Q 17 | Page 64

In a G.P., if $t_2 = 7$, $t_4 = 1575$, find r.

SOLUTION

Given $t_2 = 7$, $t_4 = 1575$

$t_n = arn–1$

$\therefore t_2 = ar$

$\therefore 7 = ar$

$a = \frac{7}{r}$ ...(i)

$t_4 = ar^3$

$\therefore ar^3 = 1575$

$\therefore r^3 \times \left(\frac{7}{r}\right) = 1575$ ...[From (i)]

$\therefore\ r^2 \times 7 = 1575$

$\therefore\ r^2 = \frac{1575}{7}$

$\therefore\ r^2 = 225$

$\therefore\ r = \pm 15.$

Miscellaneous Exercise 4 | Q 18 | Page 64

Find k so that k – 1, k, k + 2 are consecutive terms of a G.P.

SOLUTION

Since k – 1, k, k + 2 are consecutive terms of a G.P.

$\therefore\ \frac{k}{k-1} = \frac{k+2}{k}$

$\therefore\ k2 = k2 + k - 2$

$\therefore\ k - 2 = 0$

$\therefore\ k = 2.$

Miscellaneous Exercise 4 | Q 19 | Page 64

If p^{th}, q^{th} and r^{th} terms of a G.P. are x, y, z respectively, find the value of $x^{q-r} . y^{r-p} . z^{p-q}$.

SOLUTION

Let a be the first term and R be the common ratio of the G.P.

$\therefore\ t_n = a.R^{n-1}$

$\therefore\ x = a.R^{p-1},\ y = a.R^{q-1},\ z = a.R^{r-1}$

$\therefore\ x^{q-r} .y^{r-p} .z^{p-q}$

$= \left(a.R^{p-1}\right)^{q-r}.\left(a.R^{q-1}\right)^{r-p}.\left(a.R^{r-1}\right)^{p-q}$

$= a^{q-r}R^{(p-1)(q-r)} \cdot a^{r-p}R^{(q-1)(r-p)} \cdot a^{p-q}R^{(r-1)(p-q)}$

$= a^{(q-r+r-p+p-q)} \cdot R^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]}$

$= a^{0} \cdot R^{(pq-pr-q+r+qr+-pq-r+p+pr-qr-p+q)}$

$= (1).R^{0}$

$= 1.$