EXERCISE 4.1 [PAGES 50 - 51]

Exercise 4.1 | Q 1.1 | Page 50

Verify whether the following sequences are G.P. If so, write tn : 2, 6, 18, 54, ...

SOLUTION

2, 6, 18, 54, ... t1 = 2, t2 = 6, t3 = 18, t4 = 54, ... Here, $\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = 3$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here. a = 2, r = 3tn = ar^{n-1} \therefore tn = $2(3^{n-1})$

Exercise 4.1 | Q 1.2 | Page 50

Verify whether the following sequences are G.P. If so, write $t_n : 1, -5, 25, -125, ...$

SOLUTION

1, -5, 25, -125, ...
t₁ = 1, t₂ = -5, t₃ = 25, t₄ = -125, ...
Here,
$$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = -5$$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

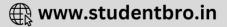
Here, a = 1, r = -5tn = arⁿ⁻¹ ∴ tn = (-5)ⁿ⁻¹

Exercise 4.1 | Q 1.3 | Page 50

Verify whether the following sequences are G.P. If so, write tn :

$$\sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, ...$$





$$\begin{split} \sqrt{5}, \frac{1}{\sqrt{5}}, \frac{1}{5\sqrt{5}}, \frac{1}{25\sqrt{5}}, \frac{1}{25\sqrt{5}}, \cdots \\ t_1 &= \sqrt{5}, t_2 = \frac{1}{\sqrt{5}}, t_3 = \frac{1}{5\sqrt{5}}, t_4 = \frac{1}{25\sqrt{5}}, \cdots \\ \text{Here,} \ \frac{t_2}{t_1} &= \frac{t_3}{t_2} = \frac{t_4}{t_3} = \frac{1}{5} \end{split}$$

Since, the ratio of any two consecutive terms is a constant, the given sequence is a geometric progression.

Here, a = $\sqrt{5}, r = rac{1}{5}$

t_n = arⁿ⁻¹
∴ tn =
$$\sqrt{5} \left(\frac{1}{5}\right)^{n-1}$$

= $(5)^{\frac{1}{2}} (5)^{1-n}$
= $(5)^{\frac{3}{2}-n}$.

Exercise 4.1 | Q 1.4 | Page 50 Verify whether the following sequences are G.P. If so, write tn : 3, 4, 5, 6, ...

SOLUTION

3, 4, 5, 6, ...

$$t_1 = 3, t_2 = 4, t_3 = 5, t_4 = 6, ...$$

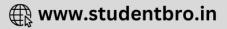
Here, $\frac{t_2}{t_1} = \frac{4}{3}, \frac{t_3}{t_2} = \frac{5}{4}, \frac{t_4}{t_3} = \frac{6}{5}$
Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

: the given sequence is not a geometric progression.

Exercise 4.1 | Q 1.5 | Page 50

Verify whether the following sequences are G.P. If so, write tn: 7, 14, 21, 28, ...





7, 14, 21, 28, ... $t_1 = 7, t_2 = 14, t_3 = 21, t_4 = 28, ...$ Here, $\frac{t_2}{t_1} = 2, \frac{t_3}{t_2} = \frac{3}{2}, \frac{t_4}{t_3} = \frac{4}{3}$ Since, $\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

.: the given sequence is not a geometric progression.

Exercise 4.1 | Q 2.1 | Page 50

For the G.P., if $r = \frac{1}{3}$, a = 9, find t_7 .

Given, $r = \frac{1}{3}$, a = 9 $t_n = ar^{n-1}$ $\therefore t_7 = 9 \times \left(\frac{1}{3}\right)^{7-1}$ $= \frac{9}{3^6}$ $= \frac{1}{81}$. Exercise 4.1 | Q 2.2 | Page 50

For the G.P., if a = $\frac{7}{243}$, $\mathbf{r} = \frac{1}{3}$, find t₃.





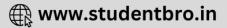
Given, a =
$$\frac{7}{243}$$
, $\mathbf{r} = \frac{1}{3}$
 $t_n = ar^{n-1}$
 $\therefore t_3 = \frac{7}{243} \times \left(\frac{1}{3}\right)^{3-1}$
 $= \frac{7}{243} \times \left(\frac{1}{3}\right)^2$
 $= \frac{7}{243} \times \frac{1}{9}$
 $= \frac{7}{2187}$.

Exercise 4.1 | Q 2.3 | Page 50 For the G.P., if a = 7, r = -3, find t₆.

SOLUTION

Given, a = 7, r = -3 $t_n = ar^{n-1}$ $\therefore t_6 = 7 \times (-3)^{6-1}$ $= 7x (-3)^5$ = 7x (-243) = -1701.Exercise 4.1 | Q 2.4 | Page 50 For the G.P., if $a = \frac{2}{3}$, $t_6 = 162$, find r. SOLUTION

Given, a =
$$\frac{2}{3}$$
, t₆ = 162



$$t_{n} = ar^{n-1}$$

$$\therefore t_{6} = \left(\frac{2}{3}\right) (r^{6-1})$$

$$\therefore 162 = \frac{2}{3}r^{5}$$

$$\therefore r^{5} = 162 \times \frac{3}{2}$$

$$\therefore r^{5} = 3^{5}$$

$$\therefore r = 3.$$

Exercise 4.1 | Q 3 | Page 50 Which term of the G. P. 5, 25, 125, 625, ... is 5¹⁰?

SOLUTION

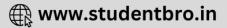
Here, $t_1 = a = 5$, $r = \frac{t_2}{t_1} = \frac{25}{5} = 5$, $t_n = 5^{10}$ $t_n = ar^{n-1}$ $\therefore 5^{10} = 5 \times 5^{(n-1)}$ $\therefore 5^{10} = 5^{(1+n-1)}$ $\therefore 5^{10} = 5^n$ $\therefore n = 10$ $\therefore 5^{10}$ is the 10th term of the G.P.

Exercise 4.1 | Q 4 | Page 50

For what values of x, $\frac{4}{3}$, x, $\frac{4}{27}$ are in G.P.?

SOLUTION

 $\frac{4}{3}, x, \frac{4}{27}$ are in geometric progression.



$$\therefore \frac{\mathbf{t}_2}{\mathbf{t}_1} = \frac{\mathbf{t}_3}{\mathbf{t}_2}$$
$$\therefore \frac{x}{\frac{4}{3}} = \frac{\frac{4}{27}}{x}$$
$$\therefore \mathbf{x}^2 = \frac{4}{3} \times \frac{4}{27}$$
$$\therefore \mathbf{x}^2 = \frac{16}{81}$$
$$\therefore \mathbf{x} = \pm \frac{4}{9}$$

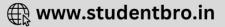
Exercise 4.1 | Q 5 | Page 50

$$\frac{5^{n-3}}{2^{n-3}}$$
,

If for a sequence, $t_n = 2^{n-3}$ so that the sequence is a G. P. Find its first term and the common ratio.

SOLUTION

The sequence (t_n) is a G.P. if $\frac{t_{n+1}}{t_n} = \text{constant for all } n \in \mathbb{N}$. Now, $t_n = \frac{5^{n-3}}{2^{n-3}}$ $\therefore t_{n+1} = \frac{5^{n+1-3}}{2^{n+1-3}} = \frac{5^{n-2}}{2^{n-2}}$ $\therefore \frac{t_{n+1}}{t_n} = \frac{5^{n-2}}{2^{n-2}} \times \frac{2^{n-3}}{5^{n-3}}$ $= (5)^{(n-2)-(n-3)} \times (2)^{(n-3)-(n-2)}$ $= (5)^1 \times (2)^{-1}$ $= \frac{5}{2}$



= constant, for all $n \in N$.

 \therefore the sequence is a G.P. with common ratio (r) = $\frac{5}{2}$

and fiist term =
$$t_1 = \frac{5^{1-3}}{2^{1-3}}$$

= $\frac{5^{-2}}{2^{-2}}$
= $\frac{2^2}{5^2}$
= $\frac{4}{25}$.

Exercise 4.1 | Q 6 | Page 51

Find three numbers in G. P. such that their sum is 21 and sum of their squares is 189.

SOLUTION

Let the three numbers in G. P. be $\displaystyle rac{\mathbf{a}}{\mathbf{r}}$, a, ar.

According to the first condition,

$$\frac{a}{r} + a + ar = 21$$

$$\therefore \frac{1}{r} + 1 + r = \frac{21}{a}$$

$$\therefore \frac{1}{r} + r = \frac{21}{a} - 1 \qquad \dots(i)$$

According to the second condition,

$$\frac{a^2}{r^2} + a^2 + a^2r^2 = 189$$

$$\therefore \frac{1}{r^2} + 1 + r^2 = \frac{189}{a^2}$$

$$\therefore \frac{1}{r^2} + r^2 = \frac{189}{a^2} - 1 \quad ...(ii)$$

On squaring equation (i), we get
$$\frac{1}{r^2} + r^2 + 2 = \frac{441}{a^2} - \frac{42}{a} + 1$$
$$\therefore \left(\frac{189}{a^2} - 1\right) + 2 = \frac{441}{a^2} - \frac{42}{a} + 1 \quad ...[From (ii)]$$
$$\therefore \frac{189}{a^2} + 1 = \frac{441}{a^2} - \frac{42}{a} + 1$$
$$\therefore 441a^2 - \frac{189}{a^2} - \frac{42}{a} = 0$$
$$\therefore \frac{252}{a^2} = \frac{42}{a}$$
$$\therefore 252 = 42a$$
$$\therefore a = 6$$

Substituting the value of a in (i), we get

$$\frac{1}{r} + r = \frac{21}{6} - 1$$

$$\therefore \frac{1+r^2}{r} = \frac{15}{6}$$

$$\therefore \frac{1+r^2}{r} = \frac{5}{2}$$

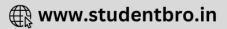
$$\therefore 2r^2 - 5r + 2 = 0$$

$$\therefore 2r^2 - 4r - r + 2 = 0$$

$$\therefore (2r - 1) (r - 2) = 0$$

$$\therefore r = \frac{1}{2} \text{ or } 2$$
When a 6, $r = \frac{1}{2}$.

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 $\frac{\mathbf{a}}{\mathbf{r}} = 12, \mathbf{a} = 6, \mathbf{ar} = 3$ When $\mathbf{a} = 6, \mathbf{r} = 2$ $\frac{\mathbf{a}}{\mathbf{r}} = 3, \mathbf{a} = 6, \mathbf{ar} = 12$ $\therefore \text{ the three numbers are } 12, 6, 3 \text{ or } 3, 6, 12.$

Exercise 4.1 | Q 7 | Page 51

Find four numbers in G. P. such that sum of the middle two numbers is 10/3 and their product is 1.

SOLUTION

Let the four numbers in G.P. be $\frac{a}{r^3}, \frac{a}{r}$, ar, ar³.

According to the second condition,

$$\frac{a}{r^3} \left(\frac{a}{r}\right) (ar) (ar^3) = 1$$

$$\therefore a^4 = 1$$

$$\therefore a = 1$$

According to the first condition,

$$\frac{a}{r} + ar \frac{10}{3}$$

$$\therefore \frac{1}{r} + (1)r = \frac{10}{3}$$

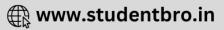
$$\therefore \frac{1+r^2}{r} = \frac{10}{3}$$

$$\therefore 3 + 3r^2 = 10r$$

$$\therefore 3r^2 - 10r + 3 = 0$$

$$\therefore (r-3)(3r-1) = 0$$

10



$$\therefore r = 3 \text{ or } r = \frac{1}{3}$$
When = 3, a = 1
$$\frac{a}{r^3} = \frac{1}{(3)^3} = \frac{1}{27}, \frac{a}{r} = \frac{1}{3}, \text{ ar } = 1(3) = 3 \text{ and } ar^3 = 1(3)^3 = 27$$
When $r = \frac{1}{3}$, $a = 1$

$$\frac{a}{r^3} = \frac{1}{(\frac{1}{3})^3} = 27, \frac{a}{r} = \frac{1}{(\frac{1}{3})} = 3,$$
ar = $1\left(\frac{1}{3}\right) = \frac{1}{3}$ and $r^3 = 1\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$$\therefore \text{ the four numbers in G.P.are}$$

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Exercise 4.1 | Q 8 | Page 51

Find five numbers in G. P. such that their product is 1024 and fifth term is square of the third term.

SOLUTION

Let the five numbers in G. P. be

$$\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$$

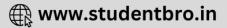
According to the given conditions,

$$\frac{\mathbf{a}}{\mathbf{r}^2} \times \frac{\mathbf{a}}{\mathbf{r}} \times \mathbf{a} \times \mathbf{ar} \times \mathbf{ar}^2 = 1024$$

$$\therefore \mathbf{a}^5 = 45$$

$$\therefore \mathbf{a} = 4 \qquad \dots(\mathbf{i})$$

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Also, $ar^2 = a^2$ $\therefore r^2 = a$ $\therefore r^2 = 4$...[From (i)] $\therefore r = \pm 2$ When a = 4, r = 2

$$rac{{
m a}}{{
m r}^2}=1, rac{{
m a}}{{
m r}}=2$$
, a = 4, ar = 8, ar² = 16

When a = 4, r = -2

$$\frac{a}{r^2} = 1, \frac{a}{r} = -2$$
, a = 4, ar = -8, ar² = 16
When a = 4, r = -2

$$rac{{
m a}}{{
m r}^2}=1, rac{{
m a}}{{
m r}}=\,-2$$
, a = 4, ar = – 8, ar 2 = 16

- ∴ the five numbers in G.P. are
- 1, 2, 4, 8, 16 or 2, 4, 8, 16.

Exercise 4.1 | Q 9 | Page 51

The fifth term of a G. P. is x, eighth term of the G. P. is y and eleventh term of the G. P. is z. Verify whether $y^2 = xz$.

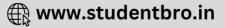
SOLUTION

Given, $t_5 = x$, $t_8 = y$, $t_{11} = z$ Since, $t_n = ar^{n-1}$ $\therefore t_5 = ar^4$, $t_8 = ar^7$, $t_{11} = ar^{10}$ Consider, L.H.S. = $y^2 = (t_8)^2 = (ar_7)^2 = a^2r^{14}$ R.H.S. = $xz = t_5.t_{11} = ar^4.ar^{10} = a^2r^{14}$ \therefore L.H.S. = R.H.S. $\therefore y^2 = xz$.

Exercise 4.1 | Q 10 | Page 51

If p, q, r, s are in G. P., show that p + q, q + r, r + s are also in G. P.





p, q, r, s are in G.P. $\therefore \frac{q}{p} = \frac{r}{q} = \frac{s}{r}$ Let $\frac{q}{p} = \frac{r}{q} = \frac{s}{r} =$ $\therefore q = pk, r = qk, s = rk$ We have to prove that p + q, q + r, r + s are in G.P. i.e. to prove that $\frac{q+r}{p+q} = \frac{r+s}{q+r}$ L.H.S. $= \frac{q+r}{p+q} = \frac{q+qk}{p+pk} \frac{q(1+k)}{p(1+k)} = \frac{q}{p} = k$ R.H.S. $= \frac{r+s}{q+r} = \frac{r+rk}{q+qk} \frac{r(1+k)}{q(1+k)} = \frac{r}{q} = k$ $\therefore \frac{q+r}{p+q} = \frac{r+s}{q+r}$ $\therefore p + q, q + r, r + s$ are in G.P.

EXERCISE 4.2 [PAGES <u>54 - 55]</u>

Exercise 4.2 | Q 1.1 | Page 54

For the following G.P.'s, find Sn: 3, 6, 12, 24, ...

1

3, 6, 12, 24, ...
Here, a = 3, r =
$$\frac{6}{3}$$
 = 2 >
 $S_n = \frac{a(r^n - 1)}{r - 1}$, for r > 1



:
$$S_n = \frac{3(2^n - 1)}{2 - 1}$$

 $\therefore S_n = 3(2^n - 1)$

Exercise 4.2 | Q 1.2 | Page 54

For the following G.P.'s, find S_n: p, q, $\frac{q^2}{p}$, $\frac{q^3}{p^2}$, ...

$$\begin{split} p, q, \frac{q^2}{p}, \frac{q^3}{p^2}, & \dots \\ \text{Here, a = p, r = } \frac{q}{p} \\ \text{Let } \frac{q}{p} < 1 \\ S_n &= \frac{a(1-r^n)}{1-r}, \text{ for } r < 1 \\ \therefore S_n &= \frac{p\left[1 - \left(\frac{q}{p}\right)^n\right]}{1 - \frac{q}{p}} \\ \therefore S_n &= \frac{p^2}{p-q} \left[1 - \left(\frac{q}{p}\right)^n\right] \\ \text{Let } \frac{q}{p} > 1 \\ \text{Let } \frac{q}{p} > 1 \\ S_n &= \frac{a(r^n - 1)}{r \ 1}, \text{ for } r > 1 \\ \therefore S_n &= \frac{p\left[\left(\frac{q}{p}\right)^n - 1\right]}{\frac{q}{p} - 1} \\ &= \frac{p^2}{q-p} \left[\left(\frac{q}{p}\right)^n - 1\right] \end{split}$$



Exercise 4.2 | Q 2.1 | Page 54

For a G.P., if a = 2, r = $-\frac{2}{3}$, find S₆. SOLUTION

a = 2, r =
$$-\frac{2}{3}$$

S_n = $\frac{a(1 - r^n)}{1 - r}$, for r < 1
∴ S₆ = $\frac{2\left[1 - \left(-\frac{2}{3}\right)^6\right]}{1 - \left(-\frac{2}{3}\right)}$
= $\frac{2\left[1 - \left(-\frac{2}{3}\right)^6\right]}{\frac{5}{3}}$
= $\frac{6}{5}\left[\frac{729 - 64}{3^6}\right]$
= $\frac{6}{5}\left[\frac{665}{729}\right]$
∴ S₆ = $\frac{266}{243}$.

Exercise 4.2 | Q 2.2 | Page 54

For a G.P., if $S_5 = 1023$, r = 4, find a.

1

$$r = 4, S_5 = 1023$$

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right), \text{ for } r >$$

$$\therefore S_5 = a\left(\frac{4^5 - 1}{4 - 1}\right)$$

$$\therefore 1023 = a\left(\frac{1024 - 1}{3}\right)$$
$$\therefore 1023 = \frac{a}{3}(1023)$$
$$\therefore a = 3.$$

Exercise 4.2 | Q 3.2 | Page 54

For a G.P., if sum of first 3 terms is 125 and sum of next 3 terms is 27, find the value of r.

$$\begin{split} &S_{3} = 125, S_{6} = 125 + 24 = 152 \\ &S_{n} = a \left(\frac{1 - r^{n}}{1 - r} \right) \\ &\therefore S_{3} = a \left(\frac{1 - r^{3}}{1 - r} \right) \\ &\therefore 125 = a \left(\frac{1 - r^{3}}{1 - r} \right) \\ &\therefore 125 = a \left(\frac{1 - r^{6}}{1 - r} \right) \\ &\therefore 152 = a \left(\frac{1 - r^{6}}{1 - r} \right) \\ &\therefore 152 = a \left(\frac{1 - r^{6}}{1 - r} \right) \\ &\text{Dividing (ii) by (i), we get} \\ \\ &\frac{152}{125} = \frac{1 - r^{6}}{1 - r^{3}} \\ &\therefore \frac{152}{125} = \frac{(1 + r^{3})(1 - r^{3})}{(1 - r^{3})} \end{split}$$

$$\therefore 1 + r^{3} = \frac{152}{125}$$
$$\therefore r^{3} = \frac{152}{125} - 1$$
$$\therefore r^{3} = \frac{27}{125}$$
$$\therefore r^{3} = \left(\frac{3}{5}\right)^{3}$$
$$\therefore r = \frac{3}{5}$$

Exercise 4.2 | Q 4.1 | Page 55

For a G.P., if t₃ = 20, t₆ = 160, find S₇.

SOLUTION

a =
$$\frac{20}{2^2}$$
 = 5
Now, S_n = $\frac{a(r^n - 1)}{r - 1}$, for > 1
∴ S₇ = $\frac{5(2^7 - 1)}{2 - 1}$
= 5(128 - 1)
= 635.

Exercise 4.2 | Q 4.2 | Page 55

For a G.P., if $t_4 = 16$, $t_9 = 512$, find S₁₀.

SOLUTION

 $t_4 = 16, t_9 = 512$ $t_n = ar^{n-1}$ ∴ $t_4 = ar^{4-1} = ar^3$ ∴ $a = \frac{16}{r^3}$...(i) Also, $t_9 = ar^8$ ∴ $ar^8 = 512$ ∴ $\frac{16}{r^3} \times r^8 = 512$ ∴ $r^5 = 32$ ∴ r = 2Substituting r = 2 in (i), we get

$$a = \frac{16}{2^3} = \frac{16}{8} = 2$$

Now, S_n =
$$\frac{a(r^n - 1)}{r - 1}$$
 for r > 1
∴ S₁₀ = $\frac{2(2^{10} - 1)}{2 - 1}$
= 2(1024 - 1)
= 2046

Exercise 4.2 | Q 5.1 | Page 55 Find the sum to n terms: 3 + 33 + 333 + 3333 + ...

SOLUTION

$$S_n = 3 + 33 + 333 + 3333 + ... \text{ upto n terms}$$

= 3(1 + 11 + 111 + ... upto n terms)
= $\frac{3}{9}$ (9 + 99 + 999 + ... upto n terms)
= $\frac{3}{9}$ [(10 - 1) + (100 - 1) + (1000 - 1) + ... upto n terms]
= $\frac{3}{9}$ [(10 + 100 + 1000 + ... upto n terms) - (1 + 1 + 1 + ... n times)]

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≫

But 10, 100,1000, ... n terms are in G.P.

With a = 10, r =
$$\frac{100}{10}$$
 = 10
 \therefore S_n = $\frac{3}{9} \left[10 \left(\frac{10^n - 1}{10 - 1} \right) - n \right]$
= $\frac{3}{9} \left[\frac{10}{9} (10^n - 1) - n \right]$
 \therefore S_n = $\frac{1}{27} [10(10^n - 1) - 9n]$

Exercise 4.2 | Q 5.2 | Page 55

Find the sum to n terms: 8 + 88 + 888 + 8888 + ...

 $S_n = 8 + 88 + 888 + \dots \text{ upto n terms}$ = 8(1 + 11 + 111 + \dots \u00ed up to n terms) = $\frac{8}{9}$ (9 + 99 + 999 + \dots \u00ed up to n terms) = $\frac{8}{9}$ [(10 - 1) + (100 - 1) + (1000 - 1) + \dots \u00ed up to terms) = $\frac{8}{9}$ [(10 + 100 + 1000 + \dots \u00ed up to terms) - (1 + 1 + 1 \dots n terms)]

But 10, 100, 1000, ... n terms are in G.P. with

a = 10, r =
$$\frac{100}{10}$$
 = 10
 \therefore S_n = $\frac{8}{9} \left[10 \left(\frac{10^{n} - 1}{10 - 1} \right) - n \right]$
= $\frac{8}{9} \left[\frac{10}{9} (10^{n} - 1) - n \right]$
 \therefore S_n = $\frac{8}{81} [10(10^{n} - 1) - 9n].$

Exercise 4.2 | Q 6.1 | Page 55

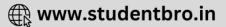
Find the sum to n term: 0.4 + 0.44 + 0.444 + ...

SOLUTION

$$S_n = 0.4 + 0.44 + 0.444 + \dots \text{ upto n terms}$$

= 4(0.1 + 0.11 + 0.111+ \dots upto n terms)
= $\frac{4}{9}$ (0.9 + 0.99 + 0.999 + \dots upto n terms)
= $\frac{4}{9}$ [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots upto n terms]





$$= \frac{4}{9} [(1 + 1 + 1 \dots n \text{ times}) - (0.01 + 0.01 + 0.001 + \dots up \text{ to } n \text{ terms})]$$

But 0.1, 0.01, 0.001, ... n terms are in G.P.

with a = 0.1, r =
$$\frac{0.01}{0.1} = 0.01$$

 \therefore S_n = $\frac{4}{9} \left\{ n - 0.1 \left[\frac{1 - (0.1)^n}{1 - 0.01} \right] \right\}$
 \therefore S_n = $\frac{4}{9} \left\{ n - \frac{0.01}{0.09} [1 - (0.1)^n] \right\}$
 \therefore S_n = $\frac{4}{9} \left[n - \frac{1}{9} (1 - (0.1)n) \right]$

Exercise 4.2 | Q 6.2 | Page 55 Find the sum to n terms: 0.7 + 0.77 + 0.777 + ...

SOLUTION

$$S_n = 0.7 + 0.77 + 0.777 + \dots \text{ upto n terms}$$

= 7(0.1 + 0.11 + 0.111 + \dots \u00ed up to n terms)
= $\frac{7}{9}$ (0.9 + 0.99 + 0.999 + \dots up to n terms)
= $\frac{7}{9}$ [(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots up to n terms]
= $\frac{7}{9}$ [(1 + 1 + 1 \dots n times) - (0.01 + 0.01 + 0.001 + \dots up to n terms)]
But 0.1, 0.01, 0.001, \dots n terms are in G.P.
with a = 0.1, r = $\frac{0.01}{0.1}$ = 0.01





$$\therefore S_{n} = \frac{7}{9} \left\{ n - 0.1 \left[\frac{1 - (0.1)^{n}}{1 - 0.01} \right] \right\}$$
$$\therefore S_{n} = \frac{7}{9} \left\{ n - \frac{0.01}{0.09} [1 - (0.1)^{n}] \right\}$$
$$\therefore S_{n} = \frac{7}{9} \left[n - \frac{1}{9} (1 - (0.1)n) \right]$$

Exercise 4.2 | Q 7.1 | Page 55

Find the nth terms of the sequences: 0.5, 0.55, 0.555, ...

SOLUTION

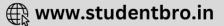
Let $t_1 = 0.5$, $t_2 = 0.55$, $t_3 = 0.555$ and so on. $t_1 = 0.5$ $t_2 = 0.55 = 0.5 + 0.05$ $t_3 = 0.555 = 0.5 + 0.05 + 0.005$ $\therefore t_n = 0.5 + 0.05 + 0.005 + ... upto n terms$ But 0.5, 0.05, 0.005, ... upto n terms are in G.P. with a = 0.5 and r = 0.1 $\therefore t_n = the sum of first n terms of the G.P.$ $\therefore t_n = 0.5 \left\{ \frac{1 - (0.1)^n}{1 - (0.1)^n} \right\}$

$$\therefore t_{n} = 0.5 \left\{ \frac{1}{1 - 0.1} \right\}$$
$$\therefore t_{n} = \frac{0.5}{0.9} \{ 1 - (0.1)^{n} \}$$
$$\therefore t_{n} = \frac{5}{9} \{ 1 - (0.1)^{n} \}$$

Exercise 4.2 | Q 7.2 | Page 55 Find the nth terms of the sequences: 0.2, 0.22, 0.222, ...

SOLUTION

Let $t_1 = 0.2$, $t_2 = 0.22$, $t_3 = 0.222$ and so on. $t_1 = 0.2$ $t_2 = 0.22 = 0.2 + 0.02$ $t_3 = 0.222 = 0.2 + 0.02 + 0.002$ \therefore $t_n = 0.2 + 0.02 + 0.002 + ...$ upto n terms But 0.2, 0.02, 0.002, ... upto n terms are in G.P. with a = 0.2 and r = 0.1 \therefore $t_n =$ the sum of first n terms of the G.P.



$$\therefore t_n = 0.2 \left\{ \frac{1 - (0.1)^n}{1 - 0.1} \right\}$$
$$\therefore t_n = \frac{0.2}{0.9} \{ 1 - (0.1)^n \}$$
$$\therefore t_n = \frac{2}{9} \{ 1 - (0.1)^n \}$$

Exercise 4.2 | Q 8 | Page 55

For a sequence, if $S_n = 2$ ($3^n - 1$), find the nth term, hence show that the sequence is a G.P.

SOLUTION

$$\begin{split} &S_n = 2(3^n - 1) \\ &\therefore S_{n-1} = 2(3^n - 1 - 1) \\ &But tn = S_n - S_{n-1} \\ &= 2(3^n - 1) - 2(3^{n-1} - 1) \\ &= 2(3^n - 1) - 2(3^{n-1} + 1) \\ &= 2(3^n - 3^{n-1}) \\ &= 2(3^{n-1+1} - 3^{n-1}) \\ &\therefore tn = 2.3^{n-1} (3 - 1) = 4.3^{n-1} \\ &\therefore t_{n+1} = 4.3^{(n+1-1)} \\ &= 4(3)^n \\ &The sequence (t_n) is a G.P. if \frac{t_{n+1}}{t_n} = constant for all n \in N. \\ &\therefore \frac{t_{n+1}}{t_n} = \frac{4(3)^n}{4(3)^{n-1}} \\ &= 3 \end{split}$$

- = constant, for all $n \in N$
- \therefore the sequence is a G.P. with $t_n = 4(3)^{n-1}$.

Exercise 4.2 | Q 9 | Page 55

If S, P, R are the sum, product and sum of the reciprocals of n terms of a G.P.

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≫

$$\left(\frac{S}{R}\right)^n = P^2.$$

respectively, then verify that

Let a be the 1st term and r be the common ratio of the G.P.

$$\therefore \text{ the G.P. is a, ar, ar^{2}, ar^{3}, ..., ar^{n-1}$$

$$\therefore S = a + ar + ar^{2} + ... + ar^{n-1} = a\left(\frac{r^{n} - 1}{r - 1}\right)$$

$$P = a(ar) (ar)^{2} ... (ar^{n-1})$$

$$= a^{n} \cdot r^{1+2+3+...+(n-1)}$$

$$= a^{n} \cdot r^{\frac{n(n-1)}{2}}$$

$$\therefore P = a^{2n} \cdot r^{n(n-1)} ...(i)$$

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^{2}} + ... + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + ... + r^{2} + r + 1}{a \cdot r^{n-1}}$$

$$= \frac{1 + r + r^{2} + ... + r^{n-2} + r^{n-1}}{a \cdot r^{n-1}}$$

1, r, r², ..., rⁿ⁻¹ are in G.P., with a = 1, r = r

$$\therefore R = \frac{1}{ar^{n-1}} \left(\frac{r^n - 1}{r - 1} \right) = \frac{1}{a^2 \cdot r^{n-1}} \times a \times \left(\frac{r^n - 1}{r - 1} \right)$$
$$\therefore R = \frac{1}{a^2 \cdot r^{n-1}} S$$
$$\therefore a^2 \cdot r^{n-1} = \frac{S}{R}$$
$$\therefore \left(a^2 \cdot r^{n-1} \right)^n = \left(\frac{S}{R} \right)^n$$



$$\begin{array}{l} \therefore \ \mathbf{a}^{2n} \cdot \mathbf{r}^{(n-1)} = \left(\frac{\mathbf{S}}{\mathbf{R}}\right)^{n} \\ \\ \therefore \ \mathsf{P}^{2} = \left(\frac{\mathbf{S}}{\mathbf{R}}\right)^{n} & \dots [\mathsf{From}\ (\mathsf{i})] \end{array}$$

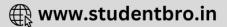
Exercise 4.2 | Q 10 | Page 55

If S_n , S_{2n} , S_{3n} are the sum of n, 2n, 3n terms of a G.P. respectively, then verify that $S_n (S_{3n} - S_{2n}) = (S_{2n} - S_n)^2$.

SOLUTION

Let a and r be the 1st term and common ratio of the G.P. respectively.

$$\begin{split} &\therefore S_n = a \left(\frac{r^n - 1}{r - 1} \right), S_{2n} = a \left(\frac{r^{2n} - 1}{r - 1} \right), S_{3n} = a \left(\frac{r^{3n} - 1}{r - 1} \right) \\ &\therefore S_{2n} - S_n = a \left(\frac{r^{2n} - 1}{r - 1} \right) - a \left(\frac{r^n - 1}{r - 1} \right) \\ &= \frac{a}{r - 1} \left(r^{2n} - 1 - r^n + 1 \right) \\ &= \frac{a}{r - 1} \left(r^{2n} - r^n \right) \\ &= \frac{ar^n}{r - 1} \left(r^n - 1 \right) \\ &\therefore S_{2n} - S_n = \frac{r^n \cdot a(r^n - 1)}{r - 1} \quad(i) \\ &S_{3n} - S_{2n} = a \left(\frac{r^{3n} - 1}{r - 1} \right) - a \left(\frac{r^{2n} - 1}{r - 1} \right) \\ &= \frac{a}{r - 1} \left(r^{3n} - 1 - r^{2n} + 1 \right) \\ &= \frac{a}{r - 1} \left(r^{3n} - r^{2n} \right) \end{split}$$



$$\begin{split} &= \frac{a}{r-1} \cdot r^{2n}(r^n - 1) \\ &= a \cdot \left(\frac{r^n - 1}{r-1}\right) \cdot r^{2n} \\ &\therefore S_n(S_{3n} - S_{2n}) = \left[a \cdot \left(\frac{r^n - 1}{r-1}\right)\right] \left[a \cdot \left(\frac{r^n - 1}{r-1}\right)r^{2n}\right] \\ &= \left[r^n \cdot \frac{a(r^n - 1)}{r-1}\right]^2 \\ &\therefore S_n(S_{3n} - S_{2n}) = (S_{2n} - S_n)^2 \quad \dots [From (i)] \\ \hline \textbf{EXERCISE 4.3 [PAGES 56 - 57]} \end{split}$$

Exercise 4.3 | Q 1.1 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

 $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \cdots$$
Here, $\mathbf{a} = \frac{1}{2}, \mathbf{r} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$
Since, $|\mathbf{r}| = \left|\frac{1}{2}\right| < 1$
 \therefore Sum to infinity exists.
Sum to infinity = $\frac{\mathbf{a}}{1-\mathbf{r}}$

$$= \frac{\frac{1}{2}}{1-\frac{1}{2}}$$
= 1.

Exercise 4.3 | Q 1.2 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

 $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$

SOLUTION

$$2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \dots$$
$$a = 2, r = \frac{\frac{4}{3}}{2} = \frac{2}{3}$$
Since, $|r| = \left|\frac{2}{3}\right| < 1$

 \therefore Sum to infinity exists.

Sum to infinity =
$$\frac{a}{1-r}$$

= $\frac{2}{1-\frac{2}{3}}$
= 6.

Exercise 4.3 | Q 1.3 | Page 56

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it:

$$-3, 1, \frac{-1}{3}, \frac{1}{9}, ...$$

$$-3, 1, \frac{-1}{3}, \frac{1}{9}, \dots$$

a = -3, r = $-\frac{1}{3}$
Since, |r| = $\left|-\frac{1}{3}\right| < 1$



.: Sum to infinity exists.

Sum to infinity =
$$\frac{a}{1-r}$$

= $\frac{-3}{1-(-\frac{1}{3})}$
= $-\frac{9}{4}$.

Exercise 4.3 | Q 1.4 | Page 57

Determine whether the sum to infinity of the following G.P'.s exist. If exists, find it $\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \cdots$

SOLUTION

$$\frac{1}{5}, \frac{-2}{5}, \frac{4}{5}, \frac{-8}{5}, \frac{16}{5}, \dots$$
$$\mathbf{a} = \frac{1}{5}, \mathbf{r} = \frac{\frac{-2}{5}}{\frac{1}{5}} = -2$$

Since, |r| = |-2| > 1

... Sum to infinity does not exist.

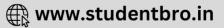
Exercise 4.3 | Q 2.1 | Page 57

Express the following recurring decimals as a rational number: $0.\overline{32}$

SOLUTION

0. $\overline{32}$ = 0.323232... = 0.32 + 0.0032 + 0.000032 + ... Here, 0.32, 0.0032, 0.000032, ... are in G.P. with a = 0.32 and r = 0.01





Since, | r | = |0.01| < 1

.: Sum to infinity exists.

$$\therefore \text{ Sum to infinity} = \frac{a}{1-r}$$
$$\therefore 0.\overline{32} = \frac{0.32}{1-(0.01)} = \frac{0.32}{0.99}$$
$$\therefore 0.\overline{32} = 32/99.$$

Exercise 4.3 | Q 2.2 | Page 57

Express the following recurring decimals as a rational number: 3.5

SOLUTION

3. 5 = 3.555... = 3 + 0.5 + 0.05 + 0.005 + ... Here, 0.5, 0.05, 0.005, ... are in G.P. with a = 0.5 and r = 0.1. Since, | r | = |0.1| < 1 ∴ Sum to infinity exists. ∴ Sum to infinity = $\frac{a}{1 - r}$ = $\frac{0.5}{1 - (0.1)}$ = $\frac{0.5}{0.9}$ = $\frac{5}{9}$ ∴ 3. 5 = 3 + $\frac{5}{9}$ = $\frac{32}{9}$.

Exercise 4.3 | Q 2.3 | Page 57

Express the following recurring decimals as a rational number: $4.\overline{18}$





 $4.\overline{18} = 4.181818...$ = 4 + 0.18 + 0.0018 + 0.000018 + ... Here, 0.18, 0.0018, 0.000018, ... are in G.P. with a = 0.18 and r = 0.01 Since, |r| = |0.01| < 1.: Sum to infinity exists. \therefore Sum to infinity = $\frac{a}{1-r}$ $=\frac{0.18}{1-(0.01)}$ $=\frac{0.18}{0.99}$ $=\frac{18}{19}$ $=\frac{2}{11}$ $\therefore 4.\overline{18} = 4 + \frac{2}{11}$ $=\frac{46}{11}$.

Exercise 4.3 | Q 2.4 | Page 57

 $0.3\overline{45}$

Express the following recurring decimals as a rational number:

SOLUTION

0.345 = 0.3454545... = 0.3 + 0.045 + 0.00045 + 0.0000045 + ... Here, 0.045, 0.00045, 0.0000045, ... are in

G.P. with a = 0.045, r = 0.01 Since, |r| = |0.01| < 1.: Sum to infinity exists. \therefore Sum to infinity = $\frac{a}{1-r}$ $=\frac{0.045}{1-0.01}$ $=\frac{0.045}{0.99}$ $=\frac{45}{990}$ $\therefore 0.3\overline{45} = 0.3 + \frac{45}{990}$ $=\frac{3}{10}+\frac{1}{22}$ $=\frac{33+5}{110}$ $=\frac{38}{110}$ $=\frac{19}{55}$ Alternate Method:

 $0.3\overline{45} = \frac{0.3\overline{45}}{10}$ = $\frac{3 + 0.45 + 0.0045 + 0.000045 + ...}{10}$ Here, 0.45, 0.0045, 0.000045... are in G.P. with a = 0.45 and r = 0.01 Since, |r| = |0.01| < 1 \therefore Sum to infinity exists. \therefore Sum to infinity = $\frac{a}{1 - r}$



=	0.45
	1 - 0.01
=	0.45
	0.99
=	45
	99
=	5
	11
÷	$0.3\overline{45} = rac{3+rac{5}{11}}{10}$
=	$\frac{38}{11}$
	$\frac{11}{10}$
=	19
	55

Exercise 4.3 | Q 2.5 | Page 57

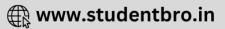
Express the following recurring decimals as a rational number: $3.4\overline{56}$

SOLUTION

3.456 = 3.4565656 ...
= 3.4 + 0.056 + 0.00056 + 0.0000056 +
Here, 0.056, 0.00056, 0.0000056, ... are in
G.P. with a = 0.056 and r = 0.01
Since,
$$|r| = |0.01| < 1$$

∴ Sum to infinity exists.
∴ Sum to infinity = $\frac{a}{1-r}$
= $\frac{0.056}{1-0.01}$

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=	0.056
	0.99
=	56
	990
÷	$3.4\overline{56} = 3.4 + rac{56}{990}$
=	34 56
	10 + 990
=	3366+56
	990
=	3422
	990
=	1711
	495

Exercise 4.3 | Q 3 | Page 57

If the common ratio of a G.P. is 2/3 and sum of its terms to infinity is 12. Find the first term.

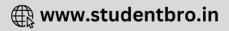
SOLUTION

r =
$$\frac{2}{3}$$
 sum to infinity = 12 ...[Given]
Sum to infinity = $\frac{a}{1 - r}$
∴ 12 = $\frac{a}{1 - \frac{2}{3}}$
∴ a = $12 \times \frac{1}{3}$
∴ a = 4.

Exercise 4.3 | Q 4 | Page 57

If the first term of a G.P. is 16 and sum of its terms to infinity is 176/5, find the common ratio.





a = 16, sum to infinity = $\frac{176}{5}$...[Given] Sum to infinity = $\frac{a}{1-r}$ $\therefore \frac{176}{5} = \frac{16}{1-r}$ $\therefore \frac{11}{5} = \frac{1}{1-r}$ $\therefore 11 - 11r = 5$ $\therefore 11r = 6$ $\therefore r = \frac{6}{11}$.

Exercise 4.3 | Q 5 | Page 57

2

The sum of the terms of an infinite G.P. is 5 and the sum of the squares of those terms is 15. Find the G.P.

SOLUTION

Let the required G.P. be a, ar, ar^2 , ar^3 , ... Sum to infinity of this G.P. = 5

$$\therefore 5 = \frac{a}{1-r}$$

$$\therefore a = 5(1-r) \qquad ...(i)$$
Also, the sum of the squares of the terms is 15.

$$\therefore (a^2 + a^2r^2 + a^2r^4 + ...) = 15$$

$$\therefore 15 = \frac{a^2}{1-r^2}$$

$$\therefore 15(1-r^2) = a^2$$

$$\therefore 15(1-r)(1+r) = 25(1-r)^2 \qquad ...[From (i)]$$

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∴ 3 (1 + r) = 5(1 - r)
∴ 3 + 3r = 5 - 5r
∴ 8r = 2
∴ r =
$$\frac{1}{4}$$

∴ a = 5 $\left(1 - \frac{1}{4}\right) = 5\left(\frac{3}{4}\right) = \frac{15}{4}$
∴ Required G.P. is a, ar, ar², ar³, ...
i.e., $\frac{15}{4}$, $\frac{15}{16}$, $\frac{15}{64}$, ...

Exercise 4.4 | Q 1.1 | Page 60

Verify whether the following sequences are H.P.: $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$, ...

SOLUTION

 $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \dots$ Here, the reciprocal sequence is 3, 5, 7, 9, ... $\therefore t_1 = 3, t_2 = 5, t_3 = 7, \dots$ $\therefore t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2, \text{ constant}$ $\therefore \text{ The reciprocal sequence is an A.P.}$ $\therefore \text{ the given sequence is H.P.}$ Exercise 4.4 | Q 1.2 | Page 60

Verify whether the following sequences are H.P. : $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{9}$, $\frac{1}{12}$, ...

SOLUTION





 $\frac{1}{3}, \frac{1}{6}, \frac{1}{9}, \frac{1}{12}, \dots$ Here, the reciprocal sequence is 3, 6, 9, 12 ... \therefore t₁ = 3, t₂ = 6, t₃ = 9, t₄ = 12, ... \therefore t₂ - t₁ = 6, t₃ = 9, t₄ = 12, ... ∴ The reciprocal sequence is an A.P.

∴ The given sequence is H.P.

Exercise 4.4 | Q 1.3 | Page 60

Verify whether the following sequences are H.P.: $\frac{1}{7}$, $\frac{1}{9}$, $\frac{1}{11}$, $\frac{1}{13}$, $\frac{1}{15}$, ...

SOLUTION

 $\frac{1}{7}, \frac{1}{9}, \frac{1}{11}, \frac{1}{13}, \frac{1}{15}, \dots$

Here, the reciprocal sequence is

 \therefore t₁ = 7, t₂ = 9, t₃ = 11, t₄ = 13, ...

 $:: t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = 2$, constant

∴ The reciprocal sequence is an A.P.

.: The given sequence is H.P.

Exercise 4.4 | Q 2.1 | Page 60

Find the nth term and hence find the 8th term of the following H.P.s: $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$

SOLUTION

 $\frac{1}{2}, \frac{1}{5}, \frac{1}{8}, \frac{1}{11}, \dots$ are in H.P. : 2, 5, 8, 11, ... are in A.P. ∴ a = 2, d = 3 tn = a + (n - 1)d= 2 + (n - 1)(3)= 3n - 1

∴ nth term of H.P. is
$$\frac{1}{3n-1}$$

∴ 8th term of H.P. = $\frac{1}{3(8)-1} = \frac{1}{23}$

Exercise 4.4 | Q 2.2 | Page 60

Find the nth term and hence find the 8th term of the following H.P.s: $\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, ...$

SOLUTION

$$\frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10}, \dots \text{ are in H.P.}$$

$$\therefore 4, 6, 8, 10, \dots \text{ are in A.P.}$$

$$\therefore a = 4, d = 2$$

$$\tan = a + (n - 1)d$$

$$= 4 + (n - 1)(2)$$

$$= 2n + 2$$

$$\therefore n^{\text{th}} \text{ term of H.P.} = \frac{1}{2n + 2}$$

$$\therefore 8^{\text{th}} \text{ term of H.P.} = \frac{1}{2(8) + 2} = \frac{1}{18}$$

Exercise 4.4 | 0.2.3 | Page 60
Find the n^{\text{th}} \text{ term and hence find the 8}^{\text{th}} \text{ term of the following H.P.s: } \frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \dots

$$\frac{1}{5}, \frac{1}{10}, \frac{1}{15}, \frac{1}{20}, \dots \text{ are in H.P.}$$

5, 10, 15, 20, ... are in H.P.
 \therefore a = 5, d = 5
 $t_n = a + (n - 1)d$

= 5 + (n − 1)(5)
= 5n
∴ nth term of H.P. =
$$\frac{1}{5n}$$

∴ 8th term of H.P. = $\frac{1}{5(8)}$
= $\frac{1}{40}$.

Exercise 4.4 | Q 3 | Page 60

Find A.M. of two positive numbers whose G.M. and H.M. are 4 and 16/5.

SOLUTION

G.M. = 4, H.M. =
$$\frac{16}{5}$$

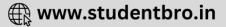
∵ (G.M.)² = (A.M.) (H.M.)
∴ 16 = A.M × $\frac{16}{5}$
∴ A.M. = 5

Exercise 4.4 | Q 4 | Page 60

Find H.M. of two positive numbers whose A.M. and G.M. are 15/2 and 6.

SOLUTION

A.M. =
$$\frac{15}{2}$$
, G.M. = 6
Now, (G.M.)² = (A.M.) (H.M.)
 $\therefore 6^2 = \frac{15}{2} \times H.M$



$$\therefore \text{ H.M.} = 36 \times \frac{2}{15}$$
$$\therefore \text{ H.M.} = \frac{24}{5}$$

Exercise 4.4 | Q 5 | Page 60

Find G.M. of two positive numbers whose A.M. and H.M. are 75 and 48.

SOLUTION

A.M. = 75, H.M. = 48 \therefore (G.M.)² = (A.M.) (H.M.) \therefore (G.M.)² = 75 x 48 = 25 x 3 x 16 x 3 = 5² x 4² x 3² \therefore G.M. = 5 x 4 x 3 \therefore G.M. = 60

Exercise 4.4 | Q 6 | Page 60

Insert two numbers between 1/7 and 1/13 so that the resulting sequence is a H.P.

SOLUTION

Let the required numbers be $\frac{1}{H_1}$ and $\frac{1}{H_2}$. $\therefore \frac{1}{7}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{13}$ are in H.P. $\therefore 7, H_1, H_2$ and 13 are in A.P. $\therefore t_1 = a = 7$ and $t_4 = a + 3d = 13$ $\therefore 7 + 3d = 13$ $\therefore 3d = 6$ $\therefore d = 2$ $\therefore H_1 = t_2 = a + d = 7 + 2 = 9$ and $H_2 = t_3 = a + 2d = 7 + 2(2) = 11$

 \div 1/9 and 1/11 are the required numbers to be inserted between 1/7 and 1/13 so that the resulting sequence is a H.P.

CLICK HERE

Exercise 4.4 | Q 7 | Page 60

Insert two numbers between 1 and -- 27 so that the resulting sequence is a G.P.

SOLUTION

Let the required numbers be G₁ and G₂. \therefore 1, G₁, G₂, -27 are in G.P. \therefore t₁ = 1, t₂ = G₁, t₃ = G₂, t₄ = -27 \therefore t₁ = a = 1 t_n = arⁿ⁻¹ \therefore t₄ = (1)⁴⁻¹ \therefore -27 = r³ \therefore r³ = (-3)³ \therefore r = -3 \therefore G₁ = t₂ = ar = 1(-3) = -3 G₂ = t₃ = ar² = 1(-3)² = 9 \therefore - 3 and 9 are the required numbers to be inse

 \div – 3 and 9 are the required numbers to be inserted between 1 and –– 27 so that th resulting sequence is a G.P.

Exercise 4.4 | Q 8 | Page 60

Find two numbers whose A.M. exceeds their G.M. by 1/2 and their H.M. by 25/26.

SOLUTION

Let a, b be the two numbers.

$$\mathsf{A} = \frac{\mathsf{a} + \mathsf{b}}{2}, \mathsf{G} = \sqrt{\mathsf{a}\mathsf{b}}, \mathsf{H} = \frac{2\mathsf{a}\mathsf{b}}{\mathsf{a} + \mathsf{b}}$$

According to the given conditions,

$$A = G + \frac{1}{2}, A = H + \frac{25}{26}$$

:..G = A - $\frac{1}{2}, H = A - \frac{25}{26}$...(i)

Now, $G^2 = AH$

$$\left(\mathbf{A}-rac{1}{2}
ight)^2=\mathbf{A}\left(\mathbf{A}-rac{25}{26}
ight)$$

 $\therefore \mathbf{A}^2-\mathbf{A}+rac{1}{4}=\mathbf{A}^2-rac{25}{26}\mathbf{A}$



 $\therefore \mathbf{A} - \frac{25}{26}\mathbf{A} = \frac{1}{4}$ $\therefore \frac{1}{26} \mathbf{A} = \frac{1}{4}$ $\therefore A = \frac{13}{2}$...(ii) $\therefore G = 6$...[From (i) and (ii)] $\therefore \frac{a+b}{2} = \frac{13}{2}$ and $\sqrt{ab} = 6$ $\therefore a + b = 13.$: b = 13 - a...(iii) and ab = 36∴ a(13 – a) = 36 ...[From (iii)] $\therefore a^2 - 13a + 36 = 0$ $\therefore (a - 4)(a - 9) = 0$ $\therefore a = 4 \text{ or } a = 9$ When a = 4, b = 13 - 4 = 9When a = 9, b = 13 - 9 = 4 \therefore the two numbers are 4 and 9.

Exercise 4.4 | Q 9 | Page 61

Find two numbers whose A.M. exceeds G.M. by 7 and their H.M. by 63/5.

SOLUTION

Let a, b be the two numbers.

$$\mathsf{A} = \frac{\mathsf{a} + \mathsf{b}}{2}, \mathsf{G} = \sqrt{\mathsf{a}\mathsf{b}}, \mathsf{H} = \frac{2\mathsf{a}\mathsf{b}}{\mathsf{a} + \mathsf{b}}$$

According to the given conditions,

 $A = G + 7, A = H + \frac{63}{5}$ \therefore G = A - 7, ...(i) $H = A - \frac{63}{5}$ Now, $G^2 = AH$ $\therefore (\mathsf{A} - 7)^2 = \mathbf{A} \left(\mathbf{A} - \frac{63}{5} \right)$ $\therefore A^2 - 14A + 49 = A^2 - \frac{63A}{5}$ $\therefore 14\mathrm{A} - \frac{63\mathrm{A}}{5} = 49$ $\therefore \frac{7A}{5} = 49$ ∴ A = 35 $\therefore \frac{a+b}{2} = 35$ ∴ a + b = 70 ∴ b = 70 – a ...(ii) ...[From (i)] G = A - 7= 35 - 7∴ G = 28 $\therefore \sqrt{ab} = 28$ $\therefore ab = 28^2 = 784$ ∴ a(70 – a) = 784 ...[From (ii)] $\therefore 70a - a^2 = 784$ $\therefore a^2 - 70a + 784$ $\therefore a^2 - 56a - 14a + 784 = 0$

∴ (a - 56) (a - 14) = 0∴ a = 14 or a = 56When a = 14, b = 70 - 14 = 14When a = 56, b = 70 - 56 = 14∴ the two numbers are 14 and 56.

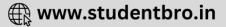
EXERCISE 4.5 [PAGE 63]

Exercise 4.5 | Q 1 | Page 63 Find the sum $\sum_{r=1}^n (r+1)(2r-1)$. SOLUTION

$$\begin{split} &\sum_{r=1}^{n} (r+1)(2r-1) \\ &= \sum_{r=1}^{n} \left(2r^2+r-1\right) \\ &= 2\sum_{r=1}^{n} r^2 + \sum_{r=1}^{n} r - \sum_{r=1}^{n} 1 \\ &= 2. \ \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - n \\ &= \frac{n}{6} \left[2(2n^2+3n+1) + 3(n+1) - 6\right] \\ &= \frac{n}{6} \left(4n^2+6n+2+3n+3-6\right) \\ &= \frac{n}{6} \left(4n^2+9n-1\right). \end{split}$$

Exercise 4.5 | Q 2 | Page 63
Find $\sum_{r=1}^{n} \left(3r^2-2r+1\right). \end{split}$

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SOLUTION

$$\begin{split} &\sum_{r=1}^{n} \left(3r^2 - 2r + 1 \right) \\ &= 3\sum_{r=1} r^2 - 2\sum_{r=1}^{n} r + \sum_{r=1} 1 \\ &= 3. \, \frac{n(n+1)(2n+1)}{6} - 2\frac{n(n+1)}{2} + n \\ &= \frac{n}{2} \left[2n^2 + 3n + 1 \right) - 2(n+1) + 2 \right] \\ &= \frac{n}{2} \left(2n^2 + 3n + 1 - 2n - 2 + 2 \right) \\ &= \frac{n}{2} \left(2n^2 + n + 1 \right). \end{split}$$

Exercise 4.5 | Q 3 | Page 63

Find
$$\sum_{r=1}^n \frac{1+2+3+...+r}{r}$$

$$\sum_{r=1}^{n} \left(\frac{1+2+3+...+r}{r} \right)$$
$$= \sum_{r=1}^{n} r \frac{(r+1)}{2r}$$
$$= \frac{1}{2} \sum_{r=1}^{n} (r+1)$$
$$= \frac{1}{2} \left[\sum_{r=1}^{n} + \sum_{r=1}^{n} 1 \right]$$

$$= \frac{1}{2} \left[\frac{n(n+1)}{2} + n \right]$$
$$= \frac{n}{4} [(n+1) + 2]$$
$$= \frac{n}{4} (n+3).$$

Exercise 4.5 | Q 4 | Page 63

Find
$$\sum_{r=1}^{n} \frac{1^3 + 2^3 + ... + r^3}{r(r+1)}$$
.

SOLUTION

We know that,

$$1^{3} + 2^{3} + 3^{3} + \dots + n^{3} = \frac{n^{2}(n+1)^{2}}{4}$$

$$\therefore 1^{3} + 2^{3} + 3^{3} + \dots + r^{3} = \frac{r^{2}(r+1)^{2}}{4}$$

$$\therefore \frac{1^{3} + 2^{3} + 3^{3} + \dots + r^{3}}{r(r+1)} = \frac{r(r+1)}{4}$$

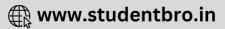
$$\therefore \sum_{r=1}^{n} \left[\frac{1^{3} + 2^{3} + 3^{3} + \dots + r^{3}}{r(r+1)} \right]$$

$$= \sum_{r=1}^{n} \frac{r(r+1)}{4}$$

$$= \frac{1}{4} \left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \right]$$

$$= \frac{1}{4} \cdot \frac{n(n+1)}{2} \left(\frac{2n+1}{3} + 1 \right)$$

$$= \frac{n(n+1)}{8} \left(\frac{2n+1+3}{3} \right)$$



$$= \frac{n(n+1)(2n+4)}{24}$$
$$= \frac{2n(n+1)(n+2)}{24}$$
$$= \frac{n(n+1)(n+2)}{12}.$$

Exercise 4.5 | Q 5 | Page 63

Find the sum $5 \times 7 + 7 \times 9 + 9 \times 11 \times 13 + \dots$ upto n terms.

SOLUTION

 $5 \times 7 + 7 \times 9 + 9 \times 11 \times 13 + \dots \text{ upto n terms}$ Now, 5, 7, 9, 11, ... are in A.P. rth term = 5 + (r - 1) (2) = 2r + 3 7, 9, 11, ... are in A.P. rth term = 7 + (r - 1) (2) = 2r + 5 $\therefore 5 \times 7 + 7 \times 9 + 9 \times 11 \times 13 + \dots$ upto n terms $= \sum_{r=1}^{n} (2r + 3)(2r + 5)$ $= \sum_{r=1}^{n} (4r^{2} + 16r + 15)$ $= 4 \sum_{r=1}^{n} r^{2} + 6 \sum_{r=1}^{n} r + 15 \sum_{r=1}^{n} 1$ $= 4 \frac{n(n+1)(2n+1)}{6} + 16 \frac{n(n+1)}{2} + 15n$ $= \frac{n}{3} [2(2n^{2} + 3n + 1) + 24(n + 1) + 45]$ $= \frac{n}{3} (4n^{2} + 6n + 2 + 24n + 24 + 45)$ $= \frac{n}{3} (4n^{2} + 30n + 71).$

Exercise 4.5 | Q 6 | Page 63

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + ...$ upto n terms.

SOLUTION

$$2^{2} + 4^{2} + 6^{2} + 8^{2} + \dots \text{ upto n terms}$$

$$= (2 \times 1)^{2} + (2 \times 2)^{2} + (2 \times 3)^{2} + (2 + x 4)^{2} + \dots$$

$$= \sum_{r=1}^{n} (2r)^{2}$$

$$= 4 \sum_{r=1}^{n} r^{2}$$

$$= \frac{4 \cdot n(n+1)(2n+1)}{6}$$

$$= \frac{2n(n+1)(2n+1)}{3}.$$

Exercise 4.5 | Q 6 | Page 63

Find the sum $2^2 + 4^2 + 6^2 + 8^2 + ...$ upto n terms.

SOLUTION

$$2^{2} + 4^{2} + 6^{2} + 8^{2} + \dots \text{ upto n terms}$$

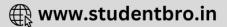
$$= (2 \times 1)^{2} + (2 \times 2)^{2} + (2 \times 3)^{2} + (2 + \times 4)^{2} + \dots$$

$$= \sum_{r=1}^{n} (2r)^{2}$$

$$= 4 \sum_{r=1}^{n} r^{2}$$

$$= \frac{4 \cdot n(n+1)(2n+1)}{6}$$

$$= \frac{2n(n+1)(2n+1)}{3}.$$
Exercise 4.5 | Q 7 | Page 63
Find $(70^{2} - 69^{2}) + (68^{2} - 67^{2}) + \dots + (2^{2} - 1^{2})$



SOLUTION

Let S = $(70^2 - 69^2) + (68^2 - 67^2) + ... + (2^2 - 1^2)$ $\therefore S = (2^2 - 1^2) + (4^2 - 3^2) + \dots + (70^2 - 69^2)$ Here, 2, 4, 6, ..., 70 is an A.P. with rth term = 2r and 1, 3, 5, ..., 69 in A.P. with rth term = 2r - 1

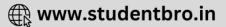
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$$\therefore S = \sum_{r=1}^{35} \left[(2r)^2 - (2r-1)^2 \right]$$
$$= \sum_{r=1}^{35} \left[4r^2 - (4r^2 - 4r + 1) \right]$$
$$= \sum_{r=1}^{35} (4r - 1)$$
$$= 4 \sum_{r=1}^{35} r - \sum_{r=1}^{35} 1$$
$$= 4. \frac{35 \times 36}{2} - 35$$
$$= (72 - 1) (35)$$
$$= 71 \times 35$$
$$= 2485.$$

Exercise 4.5 | Q 8 | Page 63 Find the sum 1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... + (2n - 1)(2n + 1)(2n + 3)

SOLUTION

 $1 \times 3 \times 5 + 3 \times 5 \times 7 + 5 \times 7 \times 9 + ... + (2n - 1)(2n + 1)(2n + 3)$ Now, 1, 3, 5, 7, ... are in A.P. with a = 1 and d = 2. \therefore rth term = 1 + (r - 1)2 = 2r - 1 3, 5, 7, 9, ... are in A.P. with a = 3 and d = 2 \therefore rth term = 3 + (r - 1)2 = 2r + 1 and 5, 7, 9, 11, ... are in A.P. with a = 5 and d = 2. \therefore rth term = 5 + (r - 1)2 = 2r + 3 \therefore 1 x 3 x 5 + 3 x 5 x 7 + 5 x 7 x 9 + ... upto n terms



$$= \sum_{r=1}^{n} (2r-1)(2r+1)(2r+3)$$

$$= \sum_{r=1}^{n} (4r^{2}-1)(2r+3)$$

$$= \sum_{r=1}^{n} (8r^{3}+12r^{2}-2r-3)$$

$$= 8\sum_{r=1}^{n} r^{3}+12\sum_{r=1}^{n} r^{2}-2\sum_{r=1}^{n} r-3\sum_{r=1}^{n} 1$$

$$= 8\left\{\frac{n(n+1)}{2}\right\}^{2}+12\left\{\frac{n(n+1)(2n+1)}{6}\right\}-2\left\{\frac{n(n+1)}{2}\right\}-3n$$

$$= 2n^{2}(n+1)^{2}+2n(n+1)(2n+1)-n(n+1)-3n$$

$$= n(n+1)[2n(n+1)+4n+2-1]-3n$$

$$= n(n+1)(2n^{2}+6n+1)-3n$$

$$= n(2n^{3}+8n^{2}+7n+1-3)$$

$$= n(2n^{3}+8n^{2}+7n+1-3)$$

$$= n(2n^{3}+8n^{2}+7n-2).$$
Exercise 4.5 [Q9] Page 63
Find n, if $\frac{1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \dots upto \ n \ terms}{1+2+3+4+\dots upto \ n \ terms} = \frac{100}{3}.$
SOLUTION

$$1 + 2 + 3 + 4 + \dots \text{upto n terms}$$

$$\therefore \frac{\sum_{r=1}^{n} r(r+1)}{\sum_{r=1}^{n} r} = \frac{100}{3}$$

$$\therefore \frac{\sum_{r=1}^{n} r^{2} + \sum_{r=1}^{n} r}{\sum_{r=1}^{n} r} = \frac{100}{3}$$

$$\therefore \frac{\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\frac{n(n+1)}{2}} = \frac{100}{3}$$
$$\therefore \frac{\frac{n(n+1)}{6} [(2n+1)+3]}{\frac{n(n+1)}{2}} = \frac{100}{3}$$
$$\therefore \frac{2(n+2)}{3} = \frac{100}{3}$$
$$\therefore n+2 = 50$$
$$\therefore n = 48.$$

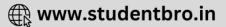
Exercise 4.5 | Q 10 | Page 63

If S₁, S₂ and S₃ are the sums of first n natural numbers, their squares and their cubes respectively, then show that: $9S_2^2 = S_3(1 + 8S_1)$.

SOLUTION

$$\begin{split} & S_{1} = 1 + 2 + 3 + ... + n = \sum_{r=1}^{n} = \frac{n(n+1)}{2} \\ & S_{2} = 1^{2} + 2^{2} + 3^{2} + ... n^{2} = \frac{n(n+1)(2n+1)}{6} \\ & S_{3} = 1^{3} + 2^{3} + 3^{3} + ... + n^{3} = \sum_{r=1}^{n} r^{3} = \frac{n^{2}(n+1)^{2}}{4} \\ & \text{R.H.S.} = S_{3}(1 + 8S_{1}) \\ & = \frac{n^{2}(n+1)^{2}}{4} \left[1 + 8 \cdot \frac{n(n+1)}{4} \right] \\ & = \frac{n^{2}(n+1)^{2}}{4} \left(1 + 4n^{2} + 4n \right) \end{split}$$





$$= \frac{n^2(n+1)^2}{4} (2n+1)^2$$
$$= \frac{9 \cdot n^2(n+1)^2 (2n+1)^2}{36}$$

MISCELLANEOUS EXERCISE 4 [PAGES 63 - 64]

Miscellaneous Exercise 4 | Q 1 | Page 63

In a G.P., the fourth term is 48 and the eighth term is 768. Find the tenth term.

SOLUTION

Given, $t_4 = 48$, $t_8 = 768$ $t_n = ar^{n-1}$ \therefore t₄ = ar³ \therefore ar³ = 48 ...(i) and ar⁷ = 768 ...(ii) Equation (ii) ÷ equation (i), we get $\frac{\mathrm{ar}^7}{\mathrm{ar}^3} = \frac{768}{48}$ \therefore r⁴ = 16 \therefore r = 2 Substituting r = 2 in (i), we get $a.(2^3) = 48$ ∴ a = 6 \therefore t₁₀ = ar⁹ \therefore t₁₀ = ar⁹ $= 6(2^9)$ = 3072.Miscellaneous Exercise 4 | Q 2 | Page 63 For a G.P. a = $rac{4}{3}$ and $t_7=rac{243}{1024}$, find the value of r. SOLUTION Given, a = $\frac{4}{3}$ and $t_7 = \frac{243}{1024}$

$$t_n = ar^{n-1}$$

$$\therefore t_7 = ar^6$$

$$\therefore \frac{243}{1024} = \frac{4}{3}r^6$$

$$\therefore r^6 = \frac{3^6}{4^6}$$

$$\therefore r = \frac{3}{4}$$

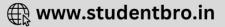
$\begin{array}{l} \mbox{Miscellaneous Exercise 4} \mid \mbox{Q 3} \mid \mbox{Page 64} \\ 5^{n-2} \end{array}$

For a sequence, if tn = $\overline{7^{n-3}}$ ' verify whether the sequence is a G.P. If it is a G.P., find its first term and the common ratio.

SOLUTION

The sequence (t_n) is a G.P. if

$$\begin{aligned} \frac{t_{n+1}}{t_n} &= \text{constant for all } n \in \mathbb{N}. \\ \text{Now, } t_n &= \frac{5^{n-2}}{7^{n-3}} \\ \therefore t_{n+1} &= \frac{5^{n+1-2}}{7^{n+1-3}} = \frac{5^{n-1}}{7^{n-2}} \\ \therefore \frac{t_{n+1}}{t_n} &= \frac{5^{n-1}}{7^{n-2}} = \frac{7^{n-3}}{5^{n-2}} \\ &= 5^{(n-1)-(n-2)} \times 7^{(n-3)-(n-2)} \\ &= 5^{(1)} \times 7^{-1} \\ &= \frac{5}{7} \end{aligned}$$



= constant , for all $n \in N$.

 \therefore the sequence is a G.P. with common ratio (r) = $\frac{5}{7}$

and first term = $t_1 = \frac{5^{1-2}}{7^{1-3}}$ = $\frac{5^{-1}}{7^{-2}}$ = $\frac{7^2}{5}$ = $\frac{49}{5}$.

Miscellaneous Exercise 4 | Q 4 | Page 64 Find three numbers in G.P., such that their sum is 35 and their product is 1000.

SOLUTION

Let the three numbers in G.P. be $\frac{a}{r}$, a, ar.

According to the first condition,

$$\frac{\mathbf{a}}{\mathbf{r}} + \mathbf{a} + \mathbf{ar} = 35$$

$$\therefore \mathbf{a} \left(\frac{1}{\mathbf{r}} + 1 + \mathbf{r} \right) = 35 \qquad \dots (i)$$

According to the second condition,

$$\left(\frac{a}{r}\right)(a)(ar) = 1000$$

 $\therefore a^3 = 1000$
 $\therefore a = 10$

Substituting the value of a in (i), we get

$$10\left(\frac{1}{r} + 1 + r\right) = 35$$

$$\therefore \frac{1}{r} + r + 1 = \frac{35}{10}$$

$$\therefore \frac{1}{r} + r = \frac{35}{10} - 1$$

$$\therefore \frac{1}{r} + r = \frac{25}{10}$$

$$\therefore \frac{1}{r} + r = \frac{5}{2}$$

$$\therefore 2r^2 - 5r + 2 = 0$$

$$\therefore (2r - 1) (r - 2) = 0$$

$$\therefore r = \frac{1}{2} \text{ or } r = 2$$

When $r = \frac{1}{2}$, $a = 10$

$$\frac{a}{r} = \frac{10}{(\frac{1}{2})} = 20, a = 10 \text{ and } ar = 10\left(\frac{1}{2}\right) = 5$$

When $r = 2$, $a = 10$

$$\frac{a}{r} = \frac{10}{2} = 5$$
, $a = 10$ and $ar = 10(2) = 20$

 \therefore the three numbers in G.P. are 20, 10, 5 or 5, 10, 20.

Miscellaneous Exercise 4 | Q 5 | Page 64

Find four numbers in G. P. such that sum of the middle two numbers is 10/3 and their product is 1.

SOLUTION

Let the four numbers in G.P. be
$$\frac{a}{r^3}$$
, $\frac{a}{r}$, ar, ar³.





According to the second condition,

$$\frac{a}{r^3} \left(\frac{a}{r}\right) (ar) (ar^3) = 1$$

$$\therefore a^4 = 1$$

$$\therefore a = 1$$

According to the first condition,

$$\frac{a}{r} + ar \frac{10}{3}$$

$$\therefore \frac{1}{r} + (1)r = \frac{10}{3}$$

$$\therefore \frac{1+r^{2}}{r} = \frac{10}{3}$$

$$\therefore 3 + 3r^{2} = 10r$$

$$\therefore 3r^{2} - 10r + 3 = 0$$

$$\therefore (r-3)(3r-1) = 0$$

$$\therefore r = 3 \text{ or } r = \frac{1}{3}$$

When $= 3, a = 1$

$$\frac{a}{r^{3}} = \frac{1}{(3)^{3}} = \frac{1}{27}, \frac{a}{r} = \frac{1}{3}, ar = 1(3) = 3 \text{ and } ar^{3} = 1(3)^{3} = 27$$

When $r = \frac{1}{3}, a = 1$

$$\frac{a}{r^{3}} = \frac{1}{(\frac{1}{3})^{3}} = 27, \frac{a}{r} = \frac{1}{(\frac{1}{3})} = 3,$$

$$ar = 1\left(\frac{1}{3}\right) = \frac{1}{3} \text{ and } r^{3} = 1\left(\frac{1}{3}\right)^{3} = \frac{1}{27}$$

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 \therefore the four numbers in G.P.are

$$\frac{1}{27}, \frac{1}{3}, 3, 27 \text{ or } 27, 3, \frac{1}{3}, \frac{1}{27}.$$

Miscellaneous Exercise 4 | Q 6 | Page 64

Find five numbers in G.P. such that their product is 243 and sum of second and fourth number is 10.

SOLUTION

Let the five numbers in G.P. be

$$\frac{\mathbf{a}}{\mathbf{r}^2}, \frac{\mathbf{a}}{\mathbf{r}}$$
, a, ar, ar².

According to the first condition,

$$\frac{\mathbf{a}}{\mathbf{r}^{2}} \times \frac{\mathbf{a}}{\mathbf{r}} \times \mathbf{a} \times \mathbf{ar} \times \mathbf{ar}^{2} = 243$$

$$\therefore \mathbf{a}^{5} = 243$$

$$\therefore \mathbf{a} = 3$$

According to the second condition,

$$\frac{\mathbf{a}}{\mathbf{r}} + \mathbf{ar} = 10$$

$$\therefore \frac{1}{\mathbf{r}} + \mathbf{r} = \frac{10}{\mathbf{a}}$$

$$\therefore \frac{1 + \mathbf{r}^{2}}{\mathbf{r}} = \frac{10}{3}$$

$$\therefore 3\mathbf{r}^{2} - 10\mathbf{r} + 3 = 0$$

$$\therefore 3\mathbf{r}^{2} - 9\mathbf{r} - \mathbf{r} + 3 = 0$$

$$\therefore (3\mathbf{r} - 1)(\mathbf{r} - 3) = 0$$

$$\therefore \mathbf{r} = \frac{1}{3}, 3$$





When a = 3, r = $\frac{1}{3}$ $\frac{a}{r^2} = 27, \frac{a}{r} = 9, a = 3, ar = 1, ar^2 = \frac{1}{3}$ When a = 3, r = 3 $\frac{a}{r^2} = \frac{1}{3}, \frac{a}{r} = 1, a = 3, ar = 9, ar^2 = \frac{1}{3}$

∴ the five numbers in G.P. are

27, 9, 3, 1,
$$\frac{1}{3}$$
 or $\frac{1}{3}$, 1, 3, 9, 27.

Miscellaneous Exercise 4 | Q 7 | Page 64

For a sequence $S_n = 4(7^n - 1)$, verify whether the sequence is a G.P.

SOLUTION

$$\begin{split} & S_n = 4(7n - 1) \\ & \therefore S_{n-1} = 4(7^{n-1} - 1) \\ & But, tn = S_n - S_{n-1} \\ & = 4(7^n - 1) - 4(7^{n-1} - 1) \\ & = 4(7^n - 1 - 7^{n-1} + 1) \\ & = 4(7^{n-1+1} - 7^{n-1}) \\ & = 4.7^{n-1} (7 - 1) \\ & \therefore tn = 27.7^{n-1} \\ & \therefore tn+1 = 24(7)^{n+1-1} \\ & = 24(7)n \end{split}$$
The sequence (t_n) is a G.P., if $\frac{t_{n+1}}{t_n} = \text{constant for all } n \in \mathbb{N}$
 $\therefore \frac{t_{n+1}}{t_n} = \frac{24(7)^n}{24(7)^{n-1}} \\ & = 7 \\ & = \text{constant, for all } n \in \mathbb{N}$
 \therefore the sequence is a G.P.

CLICK HERE

Find 2 + 22 + 222 + 2222 + ... upto n terms.

SOLUTION

$$\begin{split} &S_n = 2 + 22 + 222 + ... \text{ upto n terms} \\ &= 2(1 + 11 + 111 + ... \text{ upto n terms}) \\ &= \frac{2}{9} \left(9 + 99 + 999 + \text{upto n terms}\right) \\ &= \frac{2}{9} \left[(10 - 1) + (100 - 1) + (1000 - 1) + ... \text{ upto n terms}\right] \\ &= \frac{2}{9} \left[(10 - 1) + (100 + 100 + ... \text{ upto n terms}) - (1 + 1 + 1 ... \text{ n terms})\right] \\ &\text{Since, 10, 100,1000, ... n terms are in G.P. with a= 10, r = } \frac{100}{100} = 10 \\ &\therefore S_n = \frac{2}{9} \left[10 \left(\frac{10^n - 1}{10 - 1}\right) - n\right] \\ &= \frac{2}{9} \left[\frac{10}{9} (10^n - 1) - n\right] \end{split}$$

:
$$S_n = \frac{2}{81} [10(10^n - 1) - 9n]$$

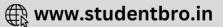
Miscellaneous Exercise 4 | Q 9 | Page 64

Find the nth term of the sequence 0.6, 0.66, 0.666, 0.6666, ...

SOLUTION

0.6, 0.66, 0.666, 0.6666, ... \therefore t₁ = 0.6 t₂ = 0.66 = 0.6 + 0.06 t₃ = 0.666 = 0.6 + 0.06 + 0.006 Hence, in general t_n = 0.6 + 0.06 + 0.006 + ...upto n terms. The terms are in G.P. with





a = 0.6, r =
$$\frac{0.06}{0.6}$$
 = 0.1

 \therefore t_n = the sum of first n terms of the G.P.

$$\begin{array}{l} \therefore \ t_n = 0.6 \left[\frac{1 - (0.1)^n}{1 - 0.1} \right] \\ = \frac{0.6}{0.9} [1 - (0.1)^n] \\ \therefore \ t_n = \frac{6}{9} [1 - (0.1)^n] \\ = \frac{2}{3} [1 - (0.1)^n]. \end{array}$$

Miscellaneous Exercise 4 | Q 10 | Page 64

Find
$$\sum_{r=1}^{n} (5r^2 + 4r - 3)$$
.

SOLUTION

$$\begin{split} &\sum_{r=1}^{n} \left(5r^{2} + 4r - 3 \right) \\ &= 5\sum_{r=1}^{n} r^{2} + 4\sum_{r=1}^{n} r - 3\sum_{r=1}^{n} 1 \\ &= 5. \frac{n(n+1)(2n+1)}{6} + 4. \frac{n(n+1)}{2} - 3n \\ &= \frac{n}{6} \left[5\left(2n^{2} + 3n + 1\right) + 12(n+1) - 18 \right] \\ &= \frac{n}{6} \left(10n^{2} + 15n + 5 + 12n + 12 - 18 \right) \\ &= \frac{n}{6} \left(10n^{2} + 27n - 1 \right). \end{split}$$

Miscellaneous Exercise 4 | Q 11 | Page 64

$$\text{Find} \sum_{r=1}^n r(r-3)(r-2).$$

SOLUTION

$$\begin{split} &\sum_{r=1}^{n} r(r-3)(r-2) \\ &= \sum_{r=1}^{n} \left(r^{3} - 5r^{2} + 6r\right) \\ &= \sum_{r=1}^{n} r^{3} - 5\sum_{r=1}^{n} r^{2} + 6\sum_{r=1}^{n} r \\ &= \frac{n^{2}(n+1)^{2}}{4} - 5\frac{n(n+1)(2n+1)}{6} + 6\frac{n(n+1)}{2} \\ &= \frac{n(n+1)}{12} \left[3n(n+1) - 10(2n+1) + 36\right] \\ &= \frac{n(n+1)}{12} \left(3n^{2} + 3n - 20n - 10 + 36\right) \\ &= \frac{n(n+1)}{12} \left(3n^{2} - 17n + 26\right). \end{split}$$

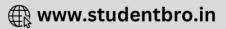
Miscellaneous Exercise 4 | Q 12 | Page 64

Find
$$\sum_{r=1}^{n} \frac{1^2 + 2^2 + 3^2 + ... r^2}{2r + 1}$$
.

SOLUTION

We know that

$$1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$$



$$\therefore 1^{2} + 2^{2} + 3^{2} + \dots + r^{2} = \frac{r(r+1)(2r+1)}{6}$$

$$\therefore \frac{1^{2} + 2^{2} + 3^{2} + \dots r^{2}}{2r+1} = \frac{r(r-1)}{6}$$

$$\therefore \sum_{r=1}^{n} \left(\frac{1^{2} + 2^{2} + 3^{2} + \dots r^{2}}{2r+1}\right)$$

$$= \sum_{r=1}^{n} \frac{r(r+1)}{6}$$

$$= \frac{1}{6} \sum_{r=1}^{n} (r^{2} + r)$$

$$= \frac{1}{6} \left(\sum_{r=1}^{n} r^{2} + \sum_{r=1}^{n} r\right)$$

$$= \frac{1}{6} \left(\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right)$$

$$= \frac{1}{6} \times \frac{n(n+1)}{2} \left(\frac{2n+1}{3} + 1\right)$$

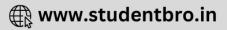
$$= \frac{n(n+1)}{12} \left(\frac{2n+1+3}{3}\right)$$

$$= \frac{n(n+1)(2n+4)}{36}$$

$$= \frac{2n(n+1)(n+2)}{36}$$

$$= \frac{n(n+1)(n+2)}{18}$$
Miscellaneous Exercise 4 [Q 13] Page 64

 $\mathsf{Find} \sum_{r=1}^{n} \frac{1^3 + 2^3 + 3^3 + ... + r^3}{\left(r+1\right)^2}$



SOLUTION

$$\begin{split} &\sum_{r=1}^{n} \frac{1^{3}+2^{3}+3^{3}+\ldots+r^{3}}{\left(r+1\right)^{2}} \\ &= \sum_{r=1}^{n} \frac{r^{2}(r+1)^{2}}{4} \times \frac{1}{\left(r+1\right)^{2}} \\ &= \frac{1}{4} \sum_{r=1}^{n} r^{2} \\ &= \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6} \\ &= \frac{n(n+1)(2n+1)}{24}. \end{split}$$

Miscellaneous Exercise 4 | Q 14 | Page 64 Find 2x + 6 + 4x 9 + 6x 12 + ... upto n terms.

SOLUTION

2, 4, 6, ... are in A.P.

$$\therefore r^{th} \text{ term } = 2 + (r - 1) 2 = 2r$$
6, 9, 12, ... are in A.P.

$$\therefore r^{th} \text{ term } = 6 + (r - 1) (3) = (3r + 3)$$

$$\therefore 2 x + 6 + 4 x 9 + 6 x 12 + ... upto n terms$$

$$= \sum_{r=1}^{n} 2r \times (3r + 3)$$

$$= 6 \sum_{r=1}^{n} r^{2} + 6 \sum_{r=1}^{n} r$$

$$= 6 \cdot \frac{n(n + 1)(2n + 1)}{6} + 6 \frac{n(n + 1)}{2}$$

$$= n(n + 1) (2n + 1 + 3)$$

$$= 2n(n + 1)(n + 2).$$

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Miscellaneous Exercise 4 | Q 15 | Page 64 Find $12^2 + 13^2 + 14^2 + 15^2 + ... + 20^2$.

SOLUTION

$$12^{2} + 13^{2} + 14^{2} + 15^{2} + ... + 20^{2}$$

$$= (1^{2} + 2^{2} + 3^{2} + 4^{2} + ... + 20^{2}) - (1^{2} + 2^{2} + 3^{2} + 4^{2} + ... + 11^{2})$$

$$= \sum_{r=1}^{20} r^{2} - \sum_{r=1}^{11} r^{2}$$

$$= \frac{20(20 + 1)(2 \times 20 + 1)}{6} - \frac{11(11 + 1)(2 \times 11 + 1)}{6}$$

$$= \frac{20 \times 21 \times 41}{6} - \frac{11 \times 12 \times 23}{6}$$

$$= 2870 - 506$$

$$= 2364.$$

Miscellaneous Exercise 4 | Q 16 | Page 64 Find $(50^2 - 49^2) + (48^2 - 47^2) + (46^2 - 45^2) + ... + (2^2 - 1^2).$

SOLUTION

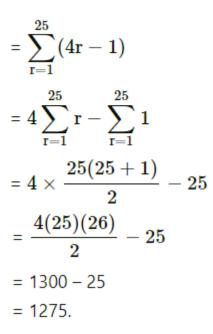
$$(50^{2} - 49^{2}) + (48^{2} - 47^{2}) + (46^{2} - 45^{2}) + ... + (2^{2} - 1^{2}).$$

$$= (50^{2} + 48^{2} + 46^{2} + ... + 2^{2}) - (49^{2} + 47^{2} + 45^{2} + ... + 1^{2})$$

$$= \sum_{r=1}^{25} (2r)^{2} - \sum_{r=1}^{25} (2r - 1)^{2}$$

$$= \sum_{r=1}^{254} 4r^{2} - \sum_{r=1}^{25} (4r^{2} - 4r + 1)$$

$$= \sum_{r=1}^{25} [4r^{2} - (4r^{2} - 4r + 1)]$$



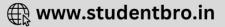
Miscellaneous Exercise 4 | Q 17 | Page 64

In a G.P., if t₂ = 7, t₄ = 1575, find r.

SOLUTION

Given
$$t_2 = 7$$
, $t_4 = 1575$
 $t_n = arn-1$
 $\therefore t_2 = ar$
 $a = \frac{7}{r}$...(i)
 $t_4 = ar^3$
 $\therefore ar^3 = 1575$
 $\therefore r^3 \times \left(\frac{7}{r}\right) = 1575$...[From (i)]





$$\therefore r^2 \ge 7 = 1575$$
$$\therefore r^2 = \frac{1575}{7}$$
$$\therefore r^2 = 225$$
$$\therefore r = \pm 15.$$

Miscellaneous Exercise 4 | Q 18 | Page 64

Find k so that k - 1, k, k + 2 are consecutive terms of a G.P.

SOLUTION

Since k - 1, k, k + 2 are consecutive terms of a G.P.

$$\therefore \frac{k}{k-1} = \frac{k+2}{k}$$
$$\therefore k^2 = k^2 + k - 2$$
$$\therefore k - 2 = 0$$
$$\therefore k = 2.$$

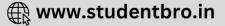
Miscellaneous Exercise 4 | Q 19 | Page 64

If p^{th} , q^{th} and r^{th} terms of a G.P. are x, y, z respectively, find the value of $x^{q-r} . y^{r-p} . z^{p-q}$.

SOLUTION

Let a be the first term and R be the common ratio of the G.P.





 $= \mathbf{a}^{q-r} \mathbf{R}^{(p-1)(q-r)} \cdot \mathbf{a}^{r-p} \mathbf{R}^{(q-1)(r-p)} \cdot \mathbf{a}^{p-q} \mathbf{R}^{(r-1)(p-q)}$ = $\mathbf{a}^{(q-r+r-p+p-q)} \cdot \mathbf{R}^{[(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)]}$ = $\mathbf{a}^{0} \cdot \mathbf{R}^{(pq-pr-q+r+qr+-pq-r+p+pr-qr-p+q)}$ = (1). \mathbf{R}^{0} = 1.



